Astro C10, Week 4: Kepler's Laws, Planets, and Tides

Who cares about Kepler?

One useful equation: $P^2 = \frac{4\pi^2}{GM}a^3$

1. When we point our telescopes in the direction of the center of the Milky Way, we stars that are moving incredibly fast. In fact, some of these stars appear to be moving in circular orbits with periods of just $\approx$1 month, at a distance of about 30 AU from some unknown central object (1 AU $\approx 1.5 \times 10^{11}$ meters).
   (a) What technique might we use to measure the velocity of this star?

   (b) Use Kepler's laws to determine the *mass* of the mystery object the star is orbiting around.

   (c) How many solar (Sun) masses is this equal to? You can use $M_\odot \approx 2 \times 10^{30}$kg.

   (d) What do you think this object is?

Star Wars: Astro C10 Edition

2. In *Star Wars: The Empire Strike Backs*[1] the Sith-ruled Empire attacks the Rebel stronghold at the Planet Hoth. Hoth, as you may recall, has 3 rocky moons and is extremely cold all year long. In fact, this permanent cold doesn't change at all; Hoth has no seasons!
   (a) What can you infer about Hoth's axis of rotation, relative to its orbital plane?

   (b) Draw the configuration involving Hoth, its 3 moons, and its Sun, that would produce the greatest tides on Hoth.

   (c) In a stunning turn of events, the Empire has re-built the Death Star! They bring it in (circular) orbit around Hoth, and prepare to fire upon the Rebels. However, the rebels launch everything they've got at the DS, forcing it to retreat to a higher orbit, at twice the original radius. By how much does the period of the orbit change? How about its speed?

[1] to be honest, I'm more of a Trekkie, but unfortunately I seem to be alone in my fandom these days; also, stay tuned for next week's edition of sci-fi themed questions: *Doctor Who*!

3. Thankfully, even if Hoth is destroyed (I'll leave you in suspense as to the outcome of this battle), the Rebels have another base that the Empire doesn't know about on Yavin IV, one of the 4 moons of the planet Yavin. Here are a few tidbits about Yavin: it is a gas giant, in a nearly circular orbit around its host star, its axis of rotation is *parallel* to its orbital plane, and it has a jungle-like climate.[2]
   (a) What would "seasons" be like on Yavin?

   (b) The Empire is scouting out Yavin. When they look at Yavin's host star for an extended period of time, they see a long dip in its brightness, with smaller symmetric drips before and after the long dip. *Draw* the brightness vs. time. What should the Empire scientists conclude about Yavin?

Other things to think about...

4. Is there a "dark side of" the moon? Explain why or why not.

5. (Bonus) We commonly use the approximation that $m_1 \gg m_2$ when applying Kepler's laws. For example, we assume the mass of the sun is much greater than the mass of the planets. This is a very useful simplifying assumption, but how valid is it?
   (a) Calculate the correction for yourself: what is the ratio of the *exact* period of Earth's orbit (i.e., when you use the full version of Kepler's law) to the period calculated from the simpler version in which we assume $M_\odot \gg M_{\text{earth}}$?

   (b) How massive must the Earth be for the calculated period to be off by a factor of, say, 10%? Compare this to the actual mass of the Earth.

6. (Bonus) I mentioned that you can derive Kepler's laws with a little basic physics. Let's try a (simplified) version of this:
   (a) For an object in a circular orbit with radius R moving at speed v, how can we relate the *period* of the orbit to the radius and velocity?
   (b) (Really bonus) Newton's law of gravity tells us that $F_G = \frac{GM_1M_2}{r^2}$. For an object in a circular orbit, the gravitational force must balance (i.e., equal) with the "centrifugal" force, $F_C = \frac{M_2v^2}{r}$. Find an expression for the speed, from these two expressions.
   (c) (Really bonus) Plug your expression from part 2 into your answer for part 1. Rearrange a little and there you have it – Kepler's 3rd law!
   (d) Why is this derivation not completely correct? *Hint: think about the assumptions we made*

[2] I may have taken some artistic license here...