1 Lots of stars

A thermal emitter of temperature T obeys the law:

$$L = A\sigma T^4 \tag{1}$$

where L is the star's luminosity, σ is a constant of proportionality and A is the surface area of the emitter. Stars are spherical thermal emitters. The area of a sphere is $A = 4\pi R^2$, which we can combine with the first equation to give $L_{\text{star}} = 4\pi R^2 \sigma T^4$.
Furthermore, the emitter's spectrum will *peak* at a wavelength λ_{max} given by:

$$\lambda_{\text{max}} = \frac{.002898m}{T}. \tag{2}$$

Hint: remember to look at ratios.

1. Star Alex is *hotter* and *brighter* than Star Nathan. Can we say which star is larger? **No, we cannot determine which star is larger. The luminosity depends on the radius *and* the temperature. We know star 1 is brighter and hotter. *Maybe* star 1 is also larger, in which case both its radius and temperature contribute to making it brighter than star 2. *Or*, maybe star 1 is smaller than star 2, but its so hot that it makes up for being smaller by a large increase in its temperature. So, in this case you can't be sure which star is larger until I give you specific values.**

2. At their respective peak wavelengths, which star (1 or 2) has shorter wavelength photons? Which star has higher frequency photons (again, at their respective peak wavelengths)? **The peak wavelength is given by Eq. ??. From this equation, we can see that as you *increase* the temperature of the star, the peak wavelength *decreases* (if this isn't clear, try plugging in a random number for T, then plug in a *bigger* random number; you should see that λ_{max} gets smaller). Since Star 1 is hotter, it has a larger T and a smaller λ_{max}. From the wave equation of light, $c = \lambda f$, we see that the larger the wavelength (λ), the smaller the frequency. Since Star 1 has a smaller λ_{max}, it must then have the *larger* frequency at the peak wavelength. Always remember – the larger the wavelength, the smaller the frequency and vice-versa.**

3. Which star has higher energy photons at their peak wavelength? **From the equation $E = hf$, we can see that the energy of a photon is directly proportional to the frequency of the photon. Since Star 1 has higher frequency photons at its peak, it will also have higher energy photons.**

4. Draw the spectrum of both stars. Think about where they should peak and the relative heights of the two curves. **I will show this in class.**

5. Describe what would you see if you looked at Star Alex through a diffraction grating. Also explain what causes the different parts of what you see. **You will see a rainbow (the "continuum" spectrum), with a few dark lines. The dark lines are absorption lines, and are caused by atoms and molecules in the cool, outer layers of the star absorbing some of the continuum light.**

6. Star Ben is twice as hot as Star Jamie but three times fainter. What is the ratio of the radii of the two stars? **From Eq. 1 we have $L = 4\pi R^2 \sigma T^4$. So Star 3 has $L_3 = 4\pi R_3^2 \sigma T_3^4$. Similarly, Star 4 has $L_4 = 4\pi R_4^2 \sigma T_4^4$. Now, let's divide these two equations:**

$$\frac{L_3}{L_4} = \frac{4\pi R_3^2 \sigma T_3^4}{4\pi R_4^2 \sigma T_4^4} \tag{3}$$

There's a factor of 4π and σ on the top and bottom, so we can cancel these. The equation simplifies to:

$$\frac{L_3}{L_4} = \frac{R_3^2 T_3^4}{R_4^2 T_4^4} \tag{4}$$

Now we have a neat expression relating the brightnesses, radii, and temperature! We don't have actual values for any of these variables, but we do have their values *in relation to on another*! I gave you that star

3 is twice as hot as star 4. Mathematically, this is simply $T_3 = 2T_4$. Likewise, I gave you that star 3 is three times fainter than star 4. Again, this is simply $L_3 = \frac{1}{3}L_4$. Let's plug these two expressions in:

$$\frac{\frac{1}{3}L_4}{L_4} = \frac{R_3^2(2T_4)^4}{R_4^2T_4^4} = \frac{R_3^2}{R_4^2}\frac{16T_4^4}{T_4^4} \tag{5}$$

We see that L_4 cancels on the left, and T_4^4 cancels on the right. So, simplifying:

$$\frac{1}{3} = \frac{16R_3^2}{R_4^2} \tag{6}$$

Remember, we're looking for the ratio, $\frac{R_3}{R_4}$. So, we divide by 16 on both sides to isolate the terms involving the radius, and then take a square root:

$$\frac{1}{3 \times 16} = \frac{R_3}{R_4} = \frac{1}{\sqrt{48}} \tag{7}$$

You can leave your answer like that, or compute $1/\sqrt{48} \approx 1/6.9$. So, star 3 has a radius about one-seventh that of star 4. That was a lot of math! This is a pretty tricky ratio problem, but if you can solve this problem you should be able to handle almost any problem involving ratios.

2 Cop chase

Not many people came to my office hours so I wandered and pulled off an armored car heist. Unfortunately, the guards called the cops who started driving in my direction. During this high-speed chase, I was plagued by a few questions, listed below. One equation you may find useful:

$$\frac{f_{\text{receiver}} - f_0}{f_0} = \frac{v_{\text{source}} - v_{\text{receiver}}}{v_{\text{wave}}} \tag{8}$$

1. As I drove off with my money, the cops close on my tail and bridging the gap between us, their sirens began to sound different. Why did I hear such a change? Give an *intuitive* explanation. Try not to use any math; draw a picture if you can.

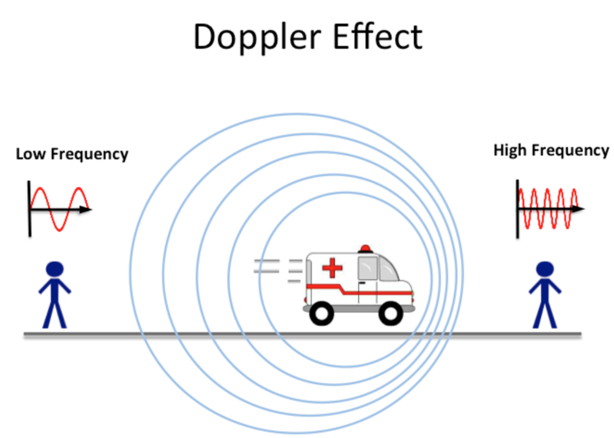

Figure 1: **The cop car (or in this picture, the ambulance) is sending out (sound) waves in my direction (I'm the observer on the right). But, since the car is moving *towards* me, these waves get pressed together. So, I'll receive one part of the wave after another *faster* than normal, which means I'll perceive a *higher* frequency. Alternatively, you can look at the space between the circles – for the observer on the right, the circles are pressed together and the wavelength is shorter. So, as the cops chase after me, I will hear a *higher* pitch (pitch is equivalent to the frequency of the sound wave) siren.**

2. I was driving at 47 m/s, and the cops at 50 m/s. How much did the frequency of the siren change? The speed of sound is 300m/s and the frequency of a police siren is 750 Hz.
   Here, we can simply plug into the formula I gave you. $v_{\text{source}} = 50m/s$ is the speed of the cop car, which is emitting the sound. $v_{\text{receiver}} = 47m/s$ is my speed. $v_{\text{wave}} = 300m/s$ is the speed of the sound wave. f_0 is the "rest," or original frequency of the siren. Solving for $f_{\text{receiver}} - f_0$ by multiplying both sides by f_0:

$$f_{\text{receiver}} - f_0 = f_0 \times \frac{v_{\text{source}} - v_{\text{receiver}}}{v_{\text{wave}}} = 750Hz \times \frac{47 - 50m/s}{300m/s} = 7.5Hz \tag{9}$$

   So the frequency *increases* – the math agrees with our intuitive explanation from part 1!

3. Is this case analogous to a "redshift" or a "blueshift"?
   When dealing with light (which is also subject to the doppler effect, since it is a wave), we call it a "blueshift" when the light wave is shifted to *smaller* wavelengths due to the doppler effect (blue light has a shorter wavelength than red light). This occurs when the source is moving *towards* the receiver (as we have here!). Likewise, in this problem we found the police siren sound wave to be doppler-shifted to higher frequencies. So, this case is analogous to a blueshift. Of course, it's not an actual blueshift since we're dealing with sound, not light.

3 For the go-getters

These are listed in order of *decreasing* relevance.

1. Do you think I was lying about my police story? Why or why not? *hint:* think about the magnitude of the frequency change.

2. Estimate the wavelength at which your body's thermal spectrum peaks. Is this in the visible range? Estimate your luminosity.

3. Consider a star moving in a circular orbit, such that at some times the star is moving directly away from us, and at others it is moving directly towards us[1]. We use the doppler shift of the star's spectrum to infer the velocity at many different times. *Sketch* what the velocity-time graph looks like.

[1] in technical terms, the orbit of the star is *edge on* relative to the observer