1 Let there be light

1. I have a beam of visible light and of radio waves. Which has a higher frequency? Higher wavelength? Higher energy? Higher speed?
   Radio has a longer wavelength (radio is the longest wavelength type, by definition). Longer wavelength always means *shorter* frequency (recall the relation $c = \lambda f$). So, visible light has the higher frequency. Higher frequency means higher energy ($E = hf = hc/\lambda$). Light always moves at a constant speed c, so they both have the same speed.

2. Suppose a gamma ray has a wavelength of 10^{-12} meters and a radio wave has a wavelength of 10 cm. What is the ratio of their energies? And their frequencies? **Recall $E = hc/\lambda$ so $E_{\text{gamma}}/E_{10\,\text{cm}} = 10^{-1}m/10^{-12}m = 10^{11}$, where $10^{-1}m = 10cm$. So the gamma ray has 10^{11} times the energy of a photon with wavelength 10cm.**

3. Calculate the frequency of a photon with $\lambda = 300$ nm. What part of the electromagnetic spectrum does this photon lie in? Would you be able to see it? Note that the speed of light is $c = 3 \times 10^8$ m/s. **Remember $c = \lambda f$ is always true. 300 nm = 300×10^{-9} m. So, $f = c/\lambda = 3 \times 10^8\, m/s / 300 \times 10^{-9} = 10^{15} Hz$. Visible light is light with wavelength roughly in the range 350 nm - 800 nm, so you wouldn't be able to see this photon.**

4. Suppose the electromagnetic spectrum extends from wavelengths of 10^{-12} meters to 10 cm. Calculate what *fraction* of all wavelengths of light the human eye can see[1].

$$\text{fraction visible} = \frac{450\text{nm}}{10\text{cm} - 10^{-12}\text{m}} \approx 3.5 \times 10^{-6}, \tag{1}$$

   where 450 nm is approximately the range of wavelengths humans can see (800 nm - 350 nm). So we can only see a super tiny portion of the whole EM spectrum!

2 Hydrogen, the essential element

When a photon – a little "packet" of light – hits an atom, it can bump one of the atom's electrons up in energy. To bump a photon up between two energy levels, the photon must have *exactly* the energy difference between the two levels. If it has too much or too little, the electron will stay put. Alternatively, the photon may have enough energy to *completely* remove the electron from the atom. This process is known as "ionization"

[1] in reality, it extends to infinitely short and long wavelengths, but this should give you a sense of how little we can see.

1. Explain the following concepts: excitation, ionization, recombination. Drawing a simple picture for each is very helpful. **Excitation: an electron in an atom gains energy, causing it to jump up to a higher energy level of the atom.**
   Ionization: an electron in an atom gains enough energy to escape the atom completely
   Recombination: a free-floating electron is reunited with an atom that's missing an electron. In doing so, the atom goes from a high energy state (free floating, unbound) to a low energy state (bound to the atom). As a result, some energy is lost, and emitted as a photon in the process.

2. For hydrogen, the energy levels are $E_n \propto -\frac{1}{n^2}$, where n is the number of the energy level. What is the ratio of the energy of an electron in the 6th energy level to one in the 1st energy level? What about the ratio of energies between the 50th and 49th energy level? $E_6/E_1 = (1/6)^2 = 1/36 = 0.027 -$ **a big difference in energies between levels 1 and 6! but,** $E_{50}/E_{44} = (44/50)^2 = 0.96$**. So, the *difference* between energy levels is small for higher energy levels.**

3. What does the above calculation tell you about the spacing of the energy levels as you go to higher levels? You can briefly sketch this by drawing a horizontal line for each energy level, with higher energy levels higher up on the page. The height difference between two horizontal lines should (roughly) correspond to the difference between those two energy levels. **see above**

4. An electron in the E_5 state of hydrogen drops down one energy level. In the process, it emits a photon. Later, this photon runs into another hydrogen atom, with an electron in some unknown energy state. Could the photon ionize the atom's electron? Could it excite the electron up to a higher energy states? If so, what transition(s) could occur? **It could excite an electron from the** $E_4 to E_5$ **state of hydrogen. It could also ionize an electron in hydrogen (but not just any electron in hydrogen! think about what condition must be met).**

5. The energy of an electron in the ground state of hydrogen is E_1. Can incoming photons with $E < E_1$ ionize an electron in the ground state? With $E = E_1$? $E > E_1$? **no, yes, yes. for *ionization* a photon needs to have energy either equal to or greater than the energy of the electron to "free" it from the atom. Any extra energy goes into the speed of the electron once its freed from the atom. On the other hand, for *excitation*, the incoming photon must have *exactly* the right energy. Be careful to remember the difference!**

3 A smattering of bonus questions

3.1 Long-haul flights

I recently flew to New Jersey, from San Francisco. The flight took 6 hours. Typical planes fly at about 500 miles per hour
What is the radius of the Earth, in kilometers? Don't look up anything. **the time difference between San Francisco and New Jersey is 3 hours. By the information given, we know that the distance between SF and NJ is 3000 miles (= 6 hours $\times$ 500 mi/hour). We also know there have to be 24 time zones across the globe. So, if three time zones makes up 3000 miles, then 24 time zones probably makes up about 24000 miles. This gives us the circumference of the Earth. Use $2\pi r = 24000$ miles to solve for r and find $r \approx 6000$km!**

3.2 Break-ups suck

After my last breakup, my friend told me not to worry: "There are plenty of fish in the sea," as the saying goes. Help me verify this by actually calculating how many fish there are in the sea[2]. Try not to look up anything. This one is harder. The answer to the previous question may be useful. **talk to me, if you're interested in the solution**

3.3 Celebrities

Estimate the total mass in lipstick that Britney Spears has used in her life. **same as above**

[2] okay, so this probably isn't what they were referring to. but oh well