

How much money can fit inside a typical armoured car?

We start by figuring out how large the truck is. Assume it's a cube, 2 meters on each side. The volume of the cube is then:

$$V_{\text{truck}} = 2m \times 2m \times 2m = 8m^3 \approx 10m^3 \quad (1)$$

Then we want to know how large (in volume) a bill is. For the long side of the bill, I estimated about half a foot, or 6 inches. For the shorter side, I guessed about 2 inches. The height is trickier, but it seems reasonable that a stack of 100 bills would be about 2 inches thick. Then, one bill would have a thickness $2/100$ inches.

Now we have enough information to compute the volume of the bill, since the volume is just the length times the width times the height. But, let's first convert to meters to keep all our units straight.

$$6\text{inches} \times \left(\frac{1\text{ft}}{12\text{inches}}\right) \times \left(\frac{1m}{3\text{ft}}\right) = \frac{1}{6}m \quad (2)$$

$$2\text{inches} \times \left(\frac{1\text{ft}}{12\text{inches}}\right) \times \left(\frac{1m}{3\text{ft}}\right) = \frac{1}{18}m \quad (3)$$

$$2/100\text{inches} \times \left(\frac{1\text{ft}}{12\text{inches}}\right) \times \left(\frac{1m}{3\text{ft}}\right) = \frac{1}{1800}m \quad (4)$$

Now, the volume of the bill is:

$$V_{\text{bill}} = \frac{1}{6} \times \frac{1}{18} \times \frac{1}{1800}m^3 \quad (5)$$

but as we discussed in section, we can simplify this a little, to avoid using a calculator:

$$V_{\text{bill}} = \frac{1}{5} \times \frac{1}{20} \times \frac{1}{2000}m^3 = \frac{1}{200000}m^3 = 5 \cdot 10^{-6}m^3 \quad (6)$$

The number of bills is just the volume of the truck divided by the volume of the bill.

$$\frac{V_{\text{truck}}}{V_{\text{bill}}} = \frac{10m^3}{5 \cdot 10^{-6}m^3} = 2 \cdot 10^6 \text{bills} \quad (7)$$

Then we just multiply by the value of each bill to get the total amount of money in the truck. I took an estimate of about 20 dollars per bill, which gives $4 \cdot 10^7$ (40 million) dollars!

In astronomy, we care about *ratios* more than actual values much of the time. In class, I gave the example of a star which obeys the proportionality law:

$$L \propto M^4 \tag{8}$$

The squiggly symbol above means “is proportional to”. Another way of interpreting this proportionality law is by understanding that what we’re *really* saying is:

$$L = kM^4 \tag{9}$$

where k is just some random number called the “constant of proportionality”. We will see that it is generally unimportant. But remember, $L \neq M^4$. The units don’t match up! The constant of proportionality helps us convert between units.

We then considered the example of 2 stars, with mass M_1 and M_2 . I told you that star 1 is twice as massive as star 2: $M_1 = 2M_2$. How much brighter is star 1 than star 2?

So, plugging into Equation 9:

$$L_1 = kM_1^4 \tag{10}$$

$$L_2 = kM_2^4 \tag{11}$$

We can divide these two equations to get:

$$\frac{L_1}{L_2} = \frac{kM_1^4}{kM_2^4} \tag{12}$$

We see that the constant of proportionality, k , cancels! So, we get:

$$\frac{L_1}{L_2} = \frac{M_1^4}{M_2^4} \tag{13}$$

Finally, we use the fact that $M_1 = 2M_2$ (remember, we’re never given the actual values of M_1 and M_2 , just their values *in relation to each other*. Plugging this in for M_1 into the above we have:

$$\frac{L_1}{L_2} = \frac{(2M_2)^4}{M_2^4} = 16 \tag{14}$$

So star 1 is 16 times as bright.