Astro C10, Quiz 1

Name:
SID:

1. I stand behind a cloud of cold hydrogen gas and shine a flashlight onto the gas. You stand on the other side of the cloud, exactly opposite me and facing the cloud. You look through a diffraction grating (a miniature-spectroscope) at the gas.

   (a) Describe, in a few sentences what you would see, and why. What kind of spectrum is this? (10 pts) **I'll see a continuous spectrum of colour (a rainbow), with a few dark "gaps" – these are absorption lines. The flashlight emits a continuous spectrum of light (it is a white light, which contains photons of all wavelengths), but the hydrogen gas absorbs certain specific wavelength photons – photons with energies corresponding to the energy differences between allowed energy levels in the hydrogen atom.**

   (b) Explain how this situation is analogous to viewing the Sun.
   The sun's atmosphere is analogous to the hydrogen gas in this case – elements present in the sun's atmosphere absorb the essentially continuous spectrum of light generated by the inner, hot parts of the star (that act like a blackbody).

2. Stars Dorian and Bryce are the same size and at the same distance, but Star Bryce is twice as hot as Star Dorian.

   (a) How much brighter does Star Bryce appear than Star Dorian?
   The formula for luminosity is given by $L = 4\pi R^2 \sigma T^4$. So, the brightness is proportional to the temperature to the fourth power. This gives:

$$\frac{L_\text{B}}{L_\text{D}} = \frac{4\pi R^2 \sigma T_B^4}{4\pi R^2 \sigma T_D^4} = \frac{T_B^4}{T_D^4} = \left(\frac{T_B}{T_D}\right)^4 = \left(\frac{2T_D}{T_D}\right)^4 = 2^4 = 16 \tag{1}$$

   where I've canceled the 4π and σ because they appear on the top and bottom; we can also cancel the R because $R_B = R_D$ since the two stars have equal radii. I also used $T_B = 2T_D$ because Bryce is twice as hot as Dorian. So, Bryce is 16 times brighter than Dorian.

(b) By what factor is the peak **wavelength** of Star Bryce longer/shorter than that of Star Dorian? Which star is bluer?
The peak wavelength is proportional to $\frac{1}{T}$. Since Bryce is *twice* as hot, its peak wavelength is *half* that of Dorian's. If you want to show this mathematically:

$$\frac{\lambda_B}{\lambda_D} = \frac{\frac{k}{T_B}}{\frac{k}{T_D}} \times \frac{T_D}{T_D} = \frac{T_D}{T_B}, \tag{2}$$

where k is Wien's constant ($= .0029$ meter-Kelvin). Plug in $T_B = 2T_D$, since Bryce is twice as hot as Dorian:

$$\frac{\lambda_B}{\lambda_D} = \frac{T_D}{2T_D} = \frac{1}{2} \tag{3}$$

So Bryce's peak wavelength is one-half that of Dorian. Since shorter wavelengths are *bluer*, Star Bryce is bluer.

3. Stars Alex and Ben are *exactly* the same, but Star Ben is double the distance from Earth of Star Alex. Suppose we use Telescope A to collect photons from Star Alex, and register 10 photons per second.

(a) How much larger must the diameter of Telescope B be to observe Star Ben and still register 10 photons per second? **If Star Ben is twice as far from Star Alex, then Ben is *four* times fainter, since the intensity of light decreases as the distance *squared*. So, we need to increase the collecting power of our telescope by a factor of *four*. The collecting power of a telescope is proportional to its area. This means that:**

$$\frac{A_B}{A_A} = 4 = \frac{D_B^2}{D_A^2} = \left(\frac{D_B}{D_A}\right)^2 \tag{4}$$

Square root both sides to find $\frac{D_B}{D_A} = 2$. So, Telescope B needs to have twice the diameter. Note that the exact number of photons per second is completely irrelevant – that phrase was just there to indicate that we want the *same* collecting power for each telescope.

(b) Which telescope will do a better job of observing a "close binary system" (a system of two stars orbiting very close to each other)? Very briefly explain why. **We use the equation:**

$$\theta_{\min} = \frac{\lambda}{D} \tag{5}$$

Since the angular resolution is *proportional* to the diameter (the smaller $\theta_{\min}$ the better your angular resolution), if you double the diameter of your telescope, you double your angular resolution. So, Telescope B has a better angular resolution by a factor of 2.

4. Consider a hypothetical atom with allowed energy levels $E_1 = 2$, $E_2 = 5$, $E_3 = 8$, $E_3 = 12$, $E_4 = 13$, and $E_5 = 15$. The ionization energy of the atom is $E = 16$. The atom's electron is in an unknown energy level.

   (a) A photon with energy $E_p = 3$ hits this atom. What transitions might occur and why? $E_1 \rightarrow E_2$, $E_2 \rightarrow E_3$. **We just need to find any two levels with an energy difference of 3.**

   (b) Could the photon ionize the atom? If so, which energy levels must the electron be in at the start? **For ionization to occur, you only need a *minimum* threshold of energy (i.e., the photon has to have enough energy to bridge the gap between the electron's current energy state and the ionization energy; you don't have to follow strict quantization rules). So, ionization can occur from E_4 and E_5, since $13 + 3 \geq 16$ and $15 + 3 \geq 16$.**

5. Who's the best GSI you've ever had? **Micah Brush for me.**

A smattering of stuff you may or may not need:

$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$
$\vec{F} = m\vec{a}$
$A_{\text{circle}} = \pi \left(\frac{D}{2}\right)^2$
$\theta = \frac{\lambda}{D}$
$V = \int\int\int dV$
$L_{\text{star}} = 4\pi R^2 \sigma T^4$