Comprehending Crazy Cosmology Crap

1. In every direction you look, you see galaxies moving away from you. There are two possible interpretations of this.

   (a) *Option 1*: A bomb went off. Everything exploded, *centered on you.* The farther bomb fragments (read: *galaxies*) are moving away faster. Explain why this solution is philosophically unsatisfying. **This violates the cosmological principle, because it would make our galaxy "special," as it would be at the center of the universe.**

   (b) *Option 2*: A slightly weirder type of bomb went off. The galaxies are fixed, but the space between them is expanding. In this scenario, explain why or why not there is a center to the universe. What does an alien civilization located in a galaxy far, far away observe? **There is no center to the universe. Even though it looks like everyone is moving away from us, an observer located in a distant galaxy will see the same thing (all galaxies appear to be moving away from them).**

2. Now that you've settled on one of these options, you go out and measure the rate of expansion (H_0); for now, let's ignore the cosmological constant (i.e., set $\Lambda = 0$).

   (a) Explain how we can use H_0 to estimate the age of the universe. $d = vt$**, but** $v = H_0 d$**, so** $d = H_0 dt \to t = 1/H_0$**. Another way to think about this is that we want to "rewind" the expansion of the universe; as we rewind the expansion, galaxies move *closer* together.** $t = 0$ **occurs when everything is all packed together. We have no information about the expansion rate in the past, so we're just going to use the expansion rate *now*, which we *can* measure**
   Also, remember – H_0 **has units of km/s/Mpc – but this is really the same as units of 1/time (because the kilometers and Mpc are both untis of distance, so you can do the unit conversion to get them to cancel).**

   (b) When is the age of the universe *exactly* equal to the expression you should have derived in the previous part? What is the value of the density parameter, Ω, in this case? **If the expansion rate is constant, then the age of the universe will *exactly* equal** $1/H_0$**. The expansion rate is only constant when there's no gravity fighting against the expansion (then it is just set by the initial conditions of the big bang). There's no gravity when there's no mass. If there's no mass, the density must be zero (density = mass/volume). If the density is zero, then** $\Omega = 0$**.**

   (c) There are two other simple cases [1]. The first is that the expansion rate of the universe has been *decreasing* over time. Is the expression you derived in part (a) an upper or lower limit for the age of the universe in this case? Explain.

   If the expansion rate has been decreasing with time, then the rate of expansion you measure now will be an underestimate for the average rate (because in the past, the expansion rate was higher – that's what it means to be decelerating). So you will compute a higher value for the age of the universe (using $t = 1/H_0$**) than the true value. Thus, your value is a *maximum*.**

   (d) The other simple case is that the expansion rate of the universe has been *increasing* over time. That is, the universe's expansion is *accelerating*. Is the expression you derived in part (a) an upper or lower limit for the age of the universe in this case? Explain. **If the expansion rate has been increasing with time, then the rate of expansion you measure now will be an overestimate for the average rate (because in the past, the expansion rate was lower). So you will compute a lower value for the age of the universe (using** $t = 1/H_0$**) than the true value. Thus, your value is a *minimum*.**

[1] Neither of which applies to our actual universe, (un)fortunately. But the result is nevertheless pedagocical. However, it turns out that $1/H_0$ is a pretty good approximation for our universe because we've had *both* a decelerating and accelerating period, and these sort of cancel out.

3. The scale factor, R, and the redshift z can be related by the simple equation $R = 1/(1+z)$. The scale factor is basically a measure of the size of the universe; if you double the scale factor, you've doubled the size of the universe, etc.

   (a) What is the scale factor at a redshift of infinity? What is the size of the universe then? And the age? What event does this correspond to? **Remember, higher redshift corresponds to earlier in the history of the universe. Thus, in an expanding universe, the the universe is smaller at higher redshift. The equation above states $R = 1/(1+z)$. As $z \to \infty$, the denominator becomes very large and you get $1/\infty$, which gives $R = 0$. This corresponds to $t = 0$, the Big Bang!**

   (b) What is the redshift now? How about the scale factor? And the age of the universe? **The redshift now is $z = 0$ (if this seems weird to you, look at anything around you; is it significantly redshifted? remember, astronomers use redshift and distance interchangeably; for things very nearby us (i.e., within the Local Group, the expansion of the universe is negligible, so there is no redshift due to the expansion). Plug into the equation to get $R = 1/(1+z) = 1/(1+0) = 1$. There's no equation for this (that we cover in this class, at least), but you should know that the age of the universe right now is about 14 billion years.**

4. A cool effect in an expanding universe is what I like to call *the matrix effect*[2]: if you shoot ten bullets at me[3], with a separation of one second between each bullet, when the bullets hit me they will be separated by *more* than one second! Mathematically, we express this as[4]:

$$\Delta t_{obs} = \Delta t_{emit}(1+z) \tag{1}$$

   (a) Suppose you are not particularly happy about your quiz grade, and you decide to take it out on me by throwing tomatoes at me. You, located at $z = 10$, throw five tomatoes at me. It takes you two seconds to pick up and throw each tomato. I'm at $z = 0$. How long do I have to dodge your tomatoes? **Use the equation, as provided. Δt_{obs} =2 seconds(1+10) = 22 seconds. So, I've got 22 seconds, or about a third of a minute, in between the arrival of each of your tomatoes – my shirt will be fine :)**

   (b) Explain how this effect implies a cosmological redshift of photons (i.e., a photon emitted far away in an expanding universe is redder by the time it reaches an observer). **Draw a wave. If the wave is moving, there will be some Δt between the arrival of the peaks of the wave at a certain point in space. This Δt is exactly equal to 1/frequency, by definition. Thus, if a photon is emitted far away from you, this Δt spacing between successive peaks of the wave will have *increased* by the time it gets to you. An increase in the period means a *decrease* in the frequency. Lower frequency (longer wavelength) corresponds to *redder* photons!**

   (c) Graph the scale factor as a function of redshift.

5. The average density of the universe is about 5 atoms per cubic meter. Compare to the density of everyday objects around you. How do you resolve the discrepancy? **The average density is...well, an average density. Yes, our particular corner of the universe is quite dense – typical densities around us are of order 1 gm/cm^3, but *most* of the universe is just empty space. This empty space brings down the average density to something incredibly low (you can get a feel for just how much of the universe is empty space by comparing 1 gm/cm^3 (density of water) to 5 atoms per cubic meter (or about 10^{-30} g/cm^3).**

6. By the way, you don't need to take my word for the fact that the critical density is equal to $3H_0^2/8\pi G$. Here's a simple way to derive it. First, let's consider a big ball of mass M of radius d. This ball may

[2] This is not a technical term...but it should be
[3] though I hope you never do this to me
[4] how can we rewrite the time-dilation equation in terms of the scale factor, R?

represent, for example, a large chunk of the universe. **this problem is a little more mathematically involved, and NOT necessary to know for the exam – but it's cool, so take a look if you're interested!**[5]

(a) The formula for kinetic energy of an object of mass m is $K = \frac{1}{2}mv^2$. To an observer at the center of our ball, what is the velocity, v, of someone of mass m at the surface of the ball (*hint:* this velocity is caused by the expansion of the universe)? Using this expression, what is the kinetic energy of a person, of mass m, sitting at the surface of the ball? Note that $m \neq M$. m is the mass of somebody sitting at the surface of our ball, M is the mass of the ball itself. $v = H_0 d$. **Plug this into KE** $= \frac{1}{2}mv^2 = \frac{1}{2}m(H_0 d)^2$.

(b) How can we rewrite M in terms of the density (call it ρ) of our ball? Remember that density = mass/volume.

$$\rho = \frac{M}{V} \rightarrow M = \rho V = \rho \frac{4}{3}\pi d^3 \tag{2}$$

I'm using V to mean volume, and I used the expression for the volume of a sphere, $\frac{4}{3}\pi d^3$.

(c) In a universe of critical density, the attractive force of gravity (pulling things together) just barely balances the expansion of the universe (pushing things apart). The expression for the gravitational energy between masses M and m is $E_g = \frac{-GMm}{d}$, and d is their separation. Substitute for M your answer to part (b). Set this equal to the expression you derived for the KE in the previous part. Solve for the density, ρ.

$$\frac{1}{2}m(H_0 d)^2 = \frac{GMm}{d} \tag{3}$$

so the m cancels.

$$\frac{1}{2}H_0^2 d^2 = \frac{GM}{d} = \frac{G\frac{4}{3}\pi d^3 \rho}{d} \tag{4}$$

Plugging in our expression for M. There's a d^2 on the left (from the KE expression) and on the right ($d^3/d = d^2$). So:

$$\frac{1}{2}H_0^2 = G\frac{4}{3}\pi\rho \tag{5}$$

Then, solve for ρ by dividing by G, multiplying by $3/(4\pi)$.

$$\rho = \frac{3H_0^2}{8\pi G} \tag{6}$$

As you can see, Alex and I are not making up these equations out of nowhere...physics works!

[5] inspiration for this problem goes to Philipp Kempski