Astro 7B – Cosmology Week 1

1. **Scale factor and redshift**
   (a) Plot the scale factor as a function of redshift.
   (b) At what redshift was the universe 1/3 of its current size?
   (c) Suppose you wanted to observe a star-forming galaxy from this epoch of the Universe. In the REST frame of that galaxy, what are the typical wavelengths of the ionizing photons it is emitting (a ballpark answer suffices). Of course, we don't observe in the rest-frame – we observe in the observer's frame! So what wavelength region should we tune our telescopes to if we want to properly observe this star-forming galaxy?
   (d) What was the temperature of the CMB at this time?

2. What is the critical density ρ_c today? Compare to the typical density of things around you (most solids – and even liquids – have densities that are comparable, to order-of-magnitude). Explain the discrepancy, given that we know we live in a flat universe today (and therefore $\Omega = 1 \rightarrow \rho = \rho_c$).

3. **Your friend Friedmann**
   (a) Use the Friedmann equation to show that for a flat universe with ONLY matter, $a(t) \propto t^{2/3}$ (you will need to integrate, but make your life easier and only worry about the scaling, i.e. drop constants). Does this setup describe our Universe? Why or why not?
   (b) Show that the Friedmann equation can be re-written as:

$$1 - \Omega(t) = \frac{-\kappa c^2}{r_0^2 a^2(t) H^2(t)}. \tag{1}$$

   This form of the Friedmann equation clearly shows that the sign of κ – i.e., the curvature/geometry of the universe – is entirely determined by Ω.
   (c) If $\Omega = 1$, how do we describe the universe: open, closed, or flat? What is the sign of κ? Sum of a triangle's angles?
   (d) If $\Omega < 1$, how do we describe the universe: open, closed, or flat? What is the sign of κ? Sum of a triangle's angles?
   (e) If $\Omega > 1$, how do we describe the universe: open, closed, or flat? What is the sign of κ? Sum of a triangle's angles?
   (f) If I tell you that $\Omega_0 = 1$, argue that $\Omega(t) = 1$ for ALL TIME. Therefore if the Universe starts off flat, it must ALWAYS stay exactly flat.

4. **Star wars** Someone once remarked to me: "George Lucas knew about dark energy all along – The Force is just Jedi controlling dark energy!". In *Episode VI: Return of the Jedi*, Yoda uses The Force to raise Luke's ship out of the lake into which it had crashed.
   (a) Assuming that he was using the dark energy within the ship to raise the ship up (i.e., converting dark energy into gravitational potential energy), how high up could he raise the ship? Thereby argue that dark energy cannot be The Force. For those unfamiliar with the scene or that need a refresher, see this link.
   (b) If we assume the Force *is* dark energy, what curvature does the star wars universe have? What Ω_Λ value would it need?