Astro 7B Worksheet – Week 8

1. **Galaxy clusters**

   (a) Typical galaxy clusters have total stellar masses (sum of masses of all stars across all galaxies in the cluster) of $\sim 10^{12} M_\odot$ and radii of ~ 1 Mpc. X-ray observations of galaxy clusters find the gas surrounding the cluster to have $kT \sim 1$ keV. Is this consistent with the stellar mass of the cluster? Why or why not?

   (b) **For fun** In the previous part you assumed that the gas is in hydrostatic equilibrium. But is this really the case? Hydrostatic equilibrium assumes there is no net energy into or out of the system. But we know the gas is cooling, by bremstrahlung. The rate at which the gas loses its thermal energy (per unit volume) is[1]:

   $$\dot{E}_{\rm cool} \approx 2 \times 10^{-27} T^{1/2} n_e n_Z, \tag{1}$$

   where n_e is the number density of electrons and n_Z is the number density of ions in the gas (remember, Bremstrahlung happens because electrons are accelerated by ions in the ionized inter-cluster plasma). Estimate (to order of magnitude) the timescale $t_{\rm cool} \sim E/\dot{E}$ (this is the timescale over which the gas loses an "order-unity" fraction of its energy – i.e., a factor of two change in energy). Assume $n_e \sim 10^{-3}$ cm^{-3}. Is our assumption of hydrostatic equilibrium reasonable?

   (c) If all galaxy clusters have constant density, how do you expect the temperature of the cluster to scale with the mass of the cluster M? A proportionality is sufficient.

2. **HL law**

   (a) You observe a galaxy with a redshift $z = 0.05$. Assume that half of the contribution to the redshift comes from the Hubble flow and half from the peculiar velocity of the galaxy, and that the galaxy moving away from us. What is the distance to the galaxy? Work in the non-relativistic limit.

   (b) Suppose the galaxy you are observing is part of a galaxy cluster, with radius $R = 1$ Mpc. What must the mass of the galaxy cluster be, to account for the peculiar velocity of this galaxy?

   (c) Show that the relativistic Doppler equation:

   $$\frac{\lambda_{\rm obs}}{\lambda_{\rm source}} = \sqrt{\frac{1+v/c}{1-v/c}} \tag{2}$$

   reduces to the classical Doppler equation:

   $$\frac{\lambda_{\rm obs} - \lambda_{\rm source}}{\lambda} \approx \frac{v}{c} \tag{3}$$

   in the appropriate limit[2].

3. **Sun-like lens** (Moaz 6.3) *A gravitational lens of mass M is halfway along the line between a source and an observer, at a distance d from each, and produces an Einstein ring.*

   (a) *Find the angular radius of the ring, θ_E, in terms of M and d.*

   (b) *If the lens is a Sun-like star with radius $R_\odot$ and mass $M_\odot$ what is the value for d at which the Einstein ring's radius appears larger than $R_\odot$?*

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[1] Deriving this expression is beyond the scope of this class; take a radiative processes class, or consult the excellent textbook by Rybicki & Lightman.

[2] In the left hand side of the denominator of the classical Doppler equation, I did not specify which λ I was referring to – $\lambda_{\rm obs}$ or $\lambda_{\rm source}$. Why doesn't it matter?