1. **IMFs** Consider an arbitrary power-law IMF $dN/dM \propto M^{-\alpha}$ extending from $0.1 M_\odot$ to infinity.
   (a) As the value of α increases, is there MORE or LESS mass locked up in higher stellar masses? In the literature, lower values of α are referred to as "top heavy" or "bottom light" – do you understand why? [1]
   (b) Give an expression for the fraction of all stars between mass M_1 and M_2.
   (c) Give an expression for the fraction of all stellar mass between M_1 and M_2. For what value of α is most of the mass locked in higher mass stars?
   (d) Give an expression for the fraction of all stellar light between M_1 and M_2. Assume that the luminosity of a star scales with its mass as $L \propto M^{\gamma}$.
   (e) Instead of talking in terms of the IMF, sometimes people talk in terms of the *cumulative* mass function (CMF). The CMF contains all the same information, but instead it is the cumulative number of stars with mass less than M, i.e. the CMF tells you $N(< M)$ (the fraction of stars "so far"). Give an expression for the CMF.
   (f) It is common to write the IMF not in terms of number of stars between M and $M + dM$, but instead in terms of *logarithmic bins*, i.e. $dN/d\log M \propto M^{\beta}$. How does β relate to α?
   (g) Given the star formation rate of the Milky Way is $\sim 1 M_\odot/\mathrm{yr}$, how many supernovae do you expect the MW to host every century?
   (h) (Bonus) How many black holes do you expect there to be in the Milky Way? Make whatever approximations you need. What is the total mass locked in black holes, if the mass of the black hole exactly equals the mass of the star from which it was produced (also definitely not true)?

2. **Metals are the new Hydrogen**. In class we derived the closed box chemical evolution model and found that gas metallicity evolves as $Z(t) = -y \ln f_g(t)$, where y is the yield (defined as the ratio of the mass in metals to that of the total mass that stays enclosed in stellar remnants).
   (a) The metallicity of the gas grows with time, as stars are made and gas is used up. Show that the mass of stars before time t, and with a metallicity less than $Z(t)$, is:

$$M_s(< Z(t)) = M_T \left[1 - \mathrm{e}^{-Z(t)/y}\right]$$

   (hint: start by rewriting $Z(t) = -y \ln f_g(t)$ as a function of $f_g(t)$. Then put $f_g(t)$ in terms of M_s)
   (b) The average metal content of the gas in the disk in the Solar neighborhood is $Z \sim 0.7 Z_\odot$, while the current mass in stars and in gas are $\sim 35\ \mathrm{M_\odot/pc^2}$ and $\sim 15\ \mathrm{M_\odot/pc^2}$, respectively. Find a value for the yield, y in the solar neighborhood assuming $Z(t = 0) = 0$.
   (c) Now let's look at the predicted number of metal-poor stars in the Solar neighborhood. Use the results from part (a) and (b) to compute the ratio of the mass in stars with 1/4 of the solar metallicity ($Z = 0.25 Z_\odot$) to the mass in stars with the current metallicity of the gas. ie) find $\frac{M_s(<0.25)}{M_s(<0.7)}$.
   (d) Compare your result from (c) to the true value of 0.02. Why do you think the result from (c) is so far off?[2]

[1] I find this terminology to be atrocioius. If you draw an IMF, the (literal) "top" of the curve you draw is where the lightest stars reside, but this is not what people mean by "top" – they mean "high mass".

[2] We call this difference in expected and true value the "G dwarf problem". The low mass stars – G dwarfs – are the stars that were formed when the galaxy had very low metallicity. So why don't we see metal-poor G-dwarfs in the local neighborhood?