Worksheet 3

1. **Really big black holes** The Event Horizon Telescope recently took the first image of a black hole. Interestingly, the EHT did not take a picture of our own – and the cloest – supermassive black hole (SMBH), Sgr A*. Instead, it took a picture of the SMBH in the galaxy M87, which is 16.4 Mpc away. However, the SMBH in M87 is more massive than that in the Milky Way, with a mass $M \approx 6 \times 10^9 M_\odot$. Compare the angular sizes of the two SMBHs. Did EHT's decision to observe M87 first make sense?

2. **Tide goes in, tide goes out...you can't explain that** Consider an object of mass m_2 and radius R_2 a distance r away from an object of mass $M_1 \gg m_2$ and radius R_1.
   (a) How does the tidal force on the secondary mass scale with distance? Compare to the scaling for the gravitational force on a point particle. Give a physical, intuitive explanation for why the tidal force scales more strongly with distance than the gravitational force.
   (b) In class we derived the tidal force by looking at the difference in gravitational forces at two distances (edge of the object being ripped apart and the center) and Taylor expanding the r^2 in the limit that the disrupting body's size was small (question: small compared to what?). Show that the tidal force $F_{\rm tid}(r)$ is therefore just equivalent to the *gradient* in the actual gravitational force, $dF_{\rm g}/dr$, across the disrupting body. This is a slightly more general way of thinking about tides.
   (c) Show that when the secondary mass reaches a distance

$$r_{\rm tid} \sim \left(\frac{\rho_1}{\rho_2}\right)^{1/3} R_1 \qquad (1)$$

   away from the primary mass that it will undergo tidal disruption. Here ρ_1 and ρ_2 are the densities for the primary and secondary, respectively. How does the shape of the disrupted object change as it approaches the tidal radius? To order of magnitude, what does the term in front of R_1 evaluate to for, say, a planet being tidally disrupted by a star? Two galaxies merging? A solar-type star being disrupted by a white dwarf? This should give you a sense for why $r_{\rm tid}$ is usually a factor of a few to a hundred greater than R_1, for a broad range of common scenarios.

3. **Stellar clusters** are often divided into two classes: so-called "open clusters" (ages $\sim$ few Myr) and "globular clusters" (ages $\sim$ 10 Gyr). Assume that all globular clusters started off as open clusters (i.e., globular clusters are just older versions of open clusters).
   (a) The oldest globular clusters have ages of $\sim$12 Gyr. What is the most massive star you would expect to find in such a cluster?
   (b) Which type of cluster is bluer?
   (c) Which has a higher mass-to-light ratio?
   (d) Draw an HR diagrams of globular and open clusters (just a qualitative sketch is sufficient). Label the various components along the HR diagram.
   (e) Which kind of cluster is more similar to the Milky Way, in terms of its constituent stars? Which kind of cluster is more similar to an elliptical galaxy?

4. (Bonus) **Tidal squeezing:** We usually think of tides as responsible for stretching things apart. But it turns out that in some cases, tidal forces can actually *squeeze* an object. This problem explores under what conditions we get compression. Consider a spherical object of mass M and radius R embedded in a larger spherical distribution of mass with density $\rho(r) \propto r^{-\alpha}$.
   (a) How does the gravitational force $GM_{\rm enc}(r)/r^2$ scale with α? We basically answered this question in our worksheet last week.
   (b) How does the tidal force scale with α? Recall that the tidal force depends on the gradient of the gravitational force $F_{\rm tid} \propto dF_g/dr$. Show that for $\alpha \geq 1$, tidal forces compress an object. Explain, physically, what is going on – why do shallow density profiles (i.e., those with sufficiently low $|\alpha|$) produce compression? This is the underlying idea behind "triggered" star formation – clouds of gas can be tidally squeezed to high enough densities that they become Jeans unstable.