Worksheet 2, Astro 7B, Spring 2021

1. **To be, Oort not to be, Part I**

   (a) Show that the Oort constants, A and B, can be written

   $$A = -\frac{1}{2}\left[\left.\frac{d\Theta}{dR}\right|_{R_0} - \frac{\Theta_0}{R_0}\right] \tag{1}$$

   $$B = -\frac{1}{2}\left[\left.\frac{d\Theta}{dR}\right|_{R_0} + \frac{\Theta_0}{R_0}\right] \tag{2}$$

   (b) Compute the quantities $A - B$ and $-(A + B)$. What does each quantity physically represent?

   (c) Use Figure 1 to determine the values of A and B at the location of the sun's orbit (use Oort's equations written in terms of A and B). Note that the y-axis is in units of km/s (not km/s/kpc).

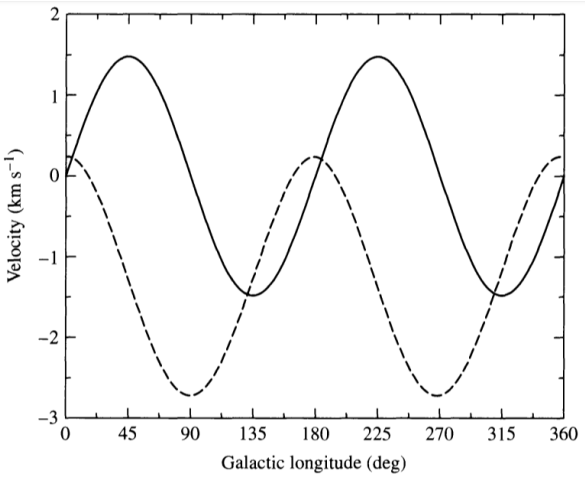

Figure 1: Best-fit curves to the velocities of stars in the solar neighborhood. The solid line is the radial velocity. The dotted line is the transverse velocity. The stars used to make this figure are all at approximately 100 pc from the sun. (C&O Fig 24.23)

   (d) Deduce ω_o and Θ_o from your answer to (c)

   (e) Deduce the circular velocity gradient in the solar neighborhood, $\left.\frac{d\Theta}{dR}\right|_{R_o}$.

   (f) Given a rigid body, what is the sign of $\left.\frac{d\Theta}{dR}\right|_{R_o}$? What does that imply about $A + B$?

   (g) What about for a flat rotation curve?

   (h) Given the values of A and B that you found from the graph above, what is the slope of the rotation curve in our local solar neighborhood?

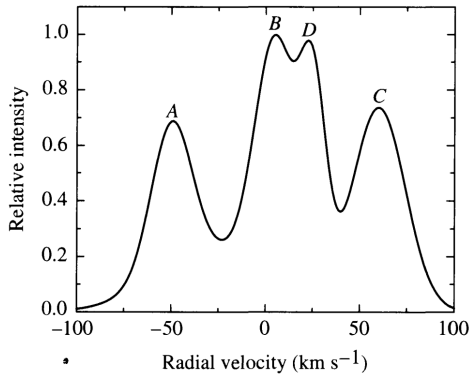

Figure 2: Observations of neutral hydrogen along a mysterious line of sight.(C&O Fig 24.24)

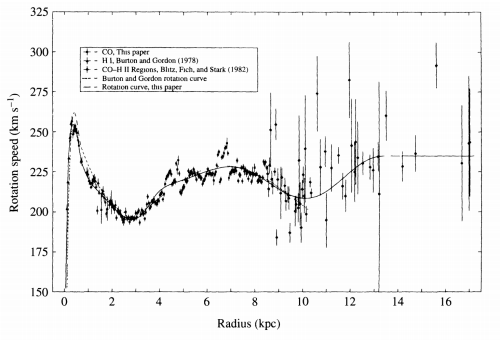

Figure 3: The rotation curve of our galaxy.(C&O Fig 24.25)

(i) This method (using the Oort constants) allows us to figure out what's going on locally. But we want to map out the rotation curve at *many* radii. What method can we use to map out $\frac{d\Theta}{dR}$ at radii *interior* to the solar galactic orbit? Why does this method only work at longitudes *interior* to the solar orbit?

(j) At what galactic longitude were the observations in Figure 2 taken? (Hint: the rotation curve of our galaxy can be found in Figure 3.)

2. **RP 19.1** A star at rest with respect to the LSR is 60° away from the solar apex. The star's parallax is $\pi''= 15$ milliarcseconds (mas). What are its radial velocity and proper motion? In what direction is the proper motion, relative to the solar apex?

3. (optional) **To be Oort or not to be, Part II**

   Take a star in a perfectly circular orbit around the galactic center at radius R. Nudge it a little, in the radial direction. The star will now undergo small *radial* oscillations about R, superimposed upon its circular motion about the galactic center. The frequency of these oscillations is known as the "epicyclic frequency"

and is given by[1]:

$$\kappa = \frac{2\Omega}{r}\frac{d}{dr}\left(r^2\Omega\right) \tag{3}$$

Show that the epicyclic frequency can be expressed in terms of Oort constants as:

$$\kappa = \sqrt{-4B(A-B)}. \tag{4}$$

Evaluate κ for the Solar neighborhood and compare to the period of the Sun's orbit around the Galactic center.

[1]The epicyclic frequency is an important concept in all of astrophysics, from planetary systems to galaxies. Take a dynamics class to learn more.