Astro 7B Discussion Worksheet 1

Spring 2021
By Nick Choksi, Andrea Antoni, and Kaela Lee

1. **Proper motions** At the center of our galaxy lies an incredibly bright radio source – Sgr A* [1] – originating from the central supermassive black hole.
   (a) Given that the distance to the center of the galaxy is $R_0 = 8$ kpc and the circular velocity at the location of the Sun is 220 km/s. What is the proper motion of Sgr A*?
   (b) Estimate how many years you would have to wait before you could detect the change in position of Sgr A* on the sky by-eye (if Sgr A* were bright enough to see by-eye). You can assume that a change in position of 1 radian is sufficient to detect the change in position.

2. **Kepler on a merry-go-round (wheeeeee)**
   (a) Draw a rotation curve ($v(r)$ versus r) for Keplerian motion and for $\omega(r) =$ constant.
   (b) Which rotation curve corresponds to our **solar system?** Which corresponds to a **merry-go-round**?
   (c) Determine how $M(r)$ scales with r for each rotation curve, assuming that gravity is making everything rotate.
   (d) Which set of curves is differentially rotating?
   (e) Take a merry-go-round initially at rest. Now spin it, but with a *Keplerian* velocity profile. What happens to the merry-go-round (a quick qualitative answer suffices)?

3. **More rotation curves** This question should help with, and give insight into, Problem 2, Homework 1. Consider a spherical mass distribution with a power-law density profile $\rho = kr^\alpha$, where k is just a "normalization" constant.
   (a) Derive an expression for the mass enclosed within radius R.
   (b) Derive an expression for the circular velocity as a function of radius.
   (c) Detailed studies find that the dark matter density profile is most precisely described by the so-called Navarro-Frenk-White (NFW) profile[2]:

$$\rho(r) = \frac{\rho_0}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2}. \tag{1}$$

   Plot $\rho(r)$ vs r/R_s using a plotting tool of your choice (ρ_0 is an arbitrary normalization). From your plot and an examination of the limits you should understand why the case $\alpha = -2$ is commonly adopted as a simple approximation of the dark matter density profile[3]. This is why the Milky Way's rotation curve is roughly flat!

[1] Pronounced "Sagittarius A-star". Because it is part of the name of an important astronomical object, this is the only horoscope name I know (and thank goodness for that!).

[2] First described in this classic paper: https://arxiv.org/abs/astro-ph/9508025

[3] In the literature, the case $\alpha = -2$ is given the fancy name "singular isothermal sphere". Why singular? Consider what happens at $r = 0$. Why isothermal? Using $\rho(r), v(r)$ and Boltzmann's constant k, construct a quantity that has units of temperature. How does the temperature scale with radius?