1 Astro 7A, Week 3

1. **Telescopes**

   (a) Consider the planned Thirty Meter Telescope (TMT) and your typical 10 inch at-home telescope. Suppose the TMT collects $N = 10^5$ photons after staring at an object for $\Delta t = 100$ seconds. How long would you have to stare with your at-home telescope to collect the same number of photons?

   (b) If I am observing with the TMT at 500 nm, what wavelength would I have to observe at with my at-home telescope to achieve a comparable diffraction limit (read: resolution)? Comment on whether this would even be possible with your at-home telescope.

   (c) Globular clusters are clusters of stars with $\sim 10^5$ stars confined to a region of ~ 1 pc. (Most) globular clusters in our galaxy live in the far reaches of the galaxy, at about $\sim$20 kpc from the Galactic center (our Solar System is located about 8 kpc from the center). Will the TMT be able to tell apart individual stars in a globular cluster in our galaxy? How about in a galaxy $d = 20$ Mpc away?

   (d) Suppose I observe a galaxy of diameter $D = 1$ kpc at a distance away $d = 1$ Mpc. If the TMT will have a plate scale of 0.06 arcseconds/pixel, over how many pixels will the galaxy image be spread out?

   (e) Stars Alex and Ben are *exactly* the same, except that Star Ben has half the diameter of Star Alex. Suppose we use Telescope A to collect photons from Star Alex, and register 10 photons per second. How much larger must the diameter of Telescope B be to observe Star Ben and still register 10 photons per second?

   (f) I put a filter on my telescope. Suppose the sensitivity function of the filter is Gaussian, centered on λ_1

   $$S_\lambda = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\lambda - \lambda_1)^2}{2\sigma^2}\right). \tag{1}$$

   If I shine light onto the telescope with a flat spectrum $F_\lambda = k$, write down an expression for the total flux through the filter.

   (g) Does the colour of the object depend on how far away I put it[1]?

2. **General Kepler**

   (a) Consider two planets with periods P_1 and P_2 (same central mass $M_\star$). What is the ratio of their semi-major axes?

   (b) Consider two planets of mass m_1 and m_2 both with the same a around the same object of mass $M_\star \gg m_1, m_2$. What is the ratio of their periods?

   (c) Derive an expression for the angular frequency ω of an object of mass m in circular orbit around $M \gg m$. And again for an elliptical orbit.

   (d) Halley's comet is a regular comet with eccentricty $e = 0.967$ and its closest approach to the Sun is 0.59 AU. What is Haley's comet's *greatest* distance from the Sun? Semi-major axis?

   (e) Show that the total energy of a Keplerian orbit can be written as:

   $$E = \left(\frac{GMm}{L}\right)^2 \frac{m}{2} \left(e^2 - 1\right) \tag{2}$$

3. **Accretion disks** Consider a little packet of mass Δm in a Keplerian orbit around a central mass M (spherical, with radius R) at radius a_i.

   (a) Write down an expression for the total orbital energy of the system.

   (b) Now suppose the little mass packet Δm experiences a little "friction" in its orbit. Does a decrease or increase over time?

[1]For the aficianados: ignore all cosmological effects.

(c) Eventually the mass packet Δm falls onto the central object (recall the central object has radius R). Write down again an expression for the *change* in orbital energy ΔE between the initial position of the mass packet and final position. Also simplify your expression by assuming $a_i \gg R$.

(d) Now suppose I have not one, but *many* little mass packets falling in over a timespan Δt. What is the resulting luminosity?

4. **Event horizon**

(a) Consider a little mass Δm around a black hole of mass M, located at radius R (the mass is NOT in orbit, it is just sitting still!). Derive the radius of the event horizon of the black hole by setting the total energy of mass m equal to zero and setting $v = c$.