1 Astro 7A, Week 2: Blackbodies

1. **Planck is your friend** Consider the Planck function in frequency space:

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/k_bT)-1}. \tag{1}$$

   (a) What are the units of B_ν? Which of the following does B_ν constitute: a luminosity, flux, intensity, flux density, specific intensity? **B_ν has units of erg/(second cm^2 Hz sr). It is a specific intensity for blackbodies.**

   (b) Derive B_λ from B_ν. **Use $B_\lambda d\lambda = B_\nu d\nu \rightarrow B_\lambda = B_\nu d\nu/d\lambda$, where $c = \lambda\nu \rightarrow d\lambda/d\nu = c/\nu^2$.**

   (c) Derive simplified expressions for B_ν in the limit of high and low-energy photons (also: what specifically do I mean by "high" and "'low" energy here? high and low energy relative to what?). **Take the limit $h\nu \gg kT$ and $h\nu \ll kT$. In the limit $h\nu \gg kT$, the -1 in the Planck function becomes negligible. Therefore $B_\nu \approx (2h\nu^3/c^2)\exp(-h\nu/kT)$. In the opposite limit $h\nu \ll kT$ then the exponential term becomes tiny. We can taylor expand $e^x \approx 1+x$ for $x \ll 1$ so that $B_\nu \approx (2\nu^2/c^2)kT$.**

   (d) Roughly sketch the spectra (on the same plot) of two blackbodies with temperatures T_1 and T_2, where $T_2 > T_1$.

2. **Stars galore**

   (a) Suppose you observe two stars, Star Dan and Star Aliza. Dan is hotter and brighter than Aliza. Can you say which star is larger? **No, we cannot determine which star is larger. The luminosity depends on the radius and the temperature. We know star 1 is brighter and hotter. Maybe star 1 is also larger, in which case both its radius and temperature contribute to making it brighter than star 2. Or, maybe star 1 is smaller than star 2, but its so hot that it makes up for being smaller by a large increase in its temperature. So, in this case you can't be sure which star is larger until I give you specific values.**

   (b) Now suppose I tell you that Dan and Aliza are at the same distance away from you. You put a *red* filter on your telescope and observe both stars. From which star will you record a greater flux? **You will receive more flux from star Dan. This tends to trip people up. Even though Aliza is cooler and therefore peaks at redder wavelengths, hotter blackbodies emit more at ALL wavelengths – i.e., the blackbody curves for two objects of different temperatures NEVER overlap.**

   (c) Star Ben is twice as hot as Star Jamie but three times fainter. If they are at the same distance, what is the ratio of the radii of the two stars?

3. **Not everything is a sphere, sadly** Consider a thin disk of gas (read: accretion disk around, e.g., a black hole). Assume the disk emits like a blackbody, and has a temperature profile $T(r) = kr^{-1/2}$ (k is a constant) from an inner disk boundary located at $R_{\rm min}$ to the outer disk edge located at $R_{\rm disk}$. Calculate the total luminosity of the disk (hint: integrate). This is an example of a problem where we cannot simply set the area of our blackbody to $A = 4\pi R^2$. But note that this isn't some contrived example just to make you suffer – accretion disks are ubiquitious in astrophysics (perhaps the second most common geometry, after spheres). **The infinitesimal luminosity is $dL = F(r)dA$. In this case, we are interested in a disk so we will integrate in terms of *rings* then $dA = 2\pi r dr$. Integrate to get the total luminosity:**

$$L = \int dL = \int F(r) 2\pi r dr. \tag{2}$$

   recall $F(r) = \sigma T^4 = \sigma k^4 r^{-2}$. Plug into the above expression and integrate away from $R_{\rm min}$ to $R_{\rm disk}$.

4. **Things moving in space I: Doppler**

(a) Consider a star moving in a circular orbit[1], such that at some times the star is moving directly away from us, and at others it is moving directly towards us[2]. We use the Doppler shift of the star's spectrum to infer the velocity at many different times. Sketch what the (observed) velocity-time graph looks like (alternatively, plot $\Delta\lambda$ as a function of time).**Sinusoidal oscillation. The peak of the sine curve will be at $+v$ and the trough at $-v$. Why sinusoidal? Because at some times the object is moving directly away from you, at other times directly towards you. And yet at other times, the motion is completely tangential and there is no radial velocity component – recall Doppler only picks out radial velocity!**

(b) Now suppose we (magically) tilt the orbital plane relative to you, the observer (I will draw this on the board). Overplot on your original sketch what the (observed) velocity-time graph would look like. **Now less of the velocity magnitude will be along the line of sight. Therefore the maximum Doppler shift you can pick up is v multiplied by a geometric factor which is less than one (so the amplitude of the sine wave decreases). Sketch a picture and see if you can work out what that geometric factor is!**

5. **Things moving in space II: Proper motions**. "Barnard's star" is the star in our night sky with the largest proper motion[3]. Since 1985, Barnard's star has moved 10.3 arcseconds/yr. Barnard's star has a parallax of 547 mas. How fast is the star moving? Also: explain why, in general, measuring full 3-D space velocities requires *a lot* of patience[4]. **Use $v_T = \mu d$. Calculate d from $d = 1/p$ – you're given the parallax. μ is just the proper motion, which is given to be 10.3 arcseconds/yr. Plug and chug to find the TANGENTIAL component of the velocity only. Why does it take so long to measure full 3D space velocities? To get the *radial* component of the velocity is quick – just take a spectrum and measure v_r from the Doppler shift (see next part). But proper motions take a long time to measure! To see this, consider Barnard's star which has the LARGEST proper motion of ANY star in the sky, and it is still only $\sim$10 arcseconds/yr. Remember that there are 206265 arcseconds in a radian – so this is still a pretty small angular shift on the sky! Most other stars have even smaller proper motions. Therefore you have to *wait* a while for the star to have an appreciable angular shift.**

6. **Things moving in space III: All together now** Finally, suppose you observe a wavelength shift in Barnard star's spectrum of $\Delta\lambda \approx 1.8$ Angstroms at $\lambda = 5000$ Angstroms. What is the full 3-D velocity of Barnard's star?. **You can calculate the radial component of the velocity v_r from the Doppler shift equation $c\Delta\lambda/\lambda = v_r$. You're given $\Delta\lambda/\lambda$ and therefore you can calculate v_r. The *total* v is given by summing in quadrature: $v = \sqrt{v_T^2 + v_r^2}$.**

[1] Perhaps the star is in a binary system and orbiting the common center of mass of the binary, as we will discuss more this week.

[2] More precisely, we say that the plane of the orbit is "edge-on" relative to the observer.

[3] Check out the wikipedia article on Barnard's star for a really cool animation.

[4] To give you a sense of this, the *Gaia* space telescope set out to measure proper motions over the course of $\sim$10 years!