1 Astro 7A, Week 2: Blackbodies

1. **Planck is your friend** Consider the Planck function in frequency space:

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/k_bT)-1}. \tag{1}$$

   (a) What are the units of B_ν? Which of the following does B_ν constitute: a luminosity, flux, intensity, flux density, specific intensity?

   (b) Derive B_λ from B_ν.

   (c) Derive simplified expressions for B_ν in the limit of high and low-energy photons (also: what specifically do I mean by "high" and "'low" energy here? high and low energy relative to what?).

   (d) Roughly sketch the spectra (on the same plot) of two blackbodies with temperatures T_1 and T_2, where $T_2 > T_1$.

2. **Stars galore**

   (a) Suppose you observe two stars, Star Dan and Star Aliza. Dan is hotter and brighter than Aliza. Can you say which star is larger?

   (b) Now suppose I tell you that Dan and Aliza are at the same distance away from you. You put a *red* filter on your telescope and observe both stars. From which star will you record a greater flux?

   (c) Star Ben is twice as hot as Star Jamie but three times fainter. If they are at the same distance, what is the ratio of the radii of the two stars?

3. **Not everything is a sphere, sadly** Consider a thin disk of gas (read: accretion disk around, e.g., a black hole). Assume the disk emits like a blackbody, and has a temperature profile $T(r) = kr^{-1/2}$ (k is a constant) from an inner disk boundary located at $R_{\rm min}$ to the outer disk edge located at $R_{\rm disk}$. Calculate the total luminosity of the disk (hint: integrate). This is an example of a problem where we cannot simply set the area of our blackbody to $A = 4\pi R^2$. But note that this isn't some contrived example just to make you suffer – accretion disks are ubiquitious in astrophysics (perhaps the second most common geometry, after spheres).

4. **Things moving in space I: Doppler**

   (a) Consider a star moving in a circular orbit[1], such that at some times the star is moving directly away from us, and at others it is moving directly towards us[2]. We use the Doppler shift of the star's spectrum to infer the velocity at many different times. Sketch what the (observed) velocity-time graph looks like (alternatively, plot $\Delta\lambda$ as a function of time).

   (b) Now suppose we (magically) tilt the orbital plane relative to you, the observer (I will draw this on the board). Overplot on your original sketch what the (observed) velocity-time graph would look like.

5. **Things moving in space II: Proper motions**. "Barnard's star" is the star in our night sky with the largest proper motion[3]. Since 1985, Barnard's star has moved 10.3 arcseconds/yr. Barnard's star has a parallax of 547 mas. How fast is the star moving? Also: explain why, in general, measuring full 3-D space velocities requires *a lot* of patience[4].

6. **Things moving in space III: All together now** Finally, suppose you observe a wavelength shift in Barnard star's spectrum of $\Delta\lambda \approx 1.8$ Angstroms at $\lambda = 5000$ Angstroms. What is the full 3-D velocity of Barnard's star?.

[1]Perhaps the star is in a binary system and orbiting the common center of mass of the binary, as we will discuss more this week.

[2]More precisely, we say that the plane of the orbit is "edge-on" relative to the observer.

[3]Check out the wikipedia article on Barnard's star for a really cool animation.

[4]To give you a sense of this, the *Gaia* space telescope set out to measure proper motions over the course of ∼10 years!