1 Astro 7A, Week 1: What the flux is a magnitude?

As a rule of thumb, astronomers like to measure everything in the most confusing way possible (because, you know, astrophysics isn't hard enough on its own). Part of this class is getting all these perverse unit systems into your blood. I hope the following questions will help you just do that.

1. Consider a star at distance d away from you. Now suppose I move it twice as far away. How much do the following change: luminosity, flux, flux density, surface brightness, intensity, specific intensity, absolute magnitude, apparent magnitude? [1]

$\boxed{\frac{M'}{M} = 1}$ $\boxed{\frac{L'}{L} = 1}$ $\boxed{\frac{F'}{F} = \frac{1}{4}}$ $\frac{F_\lambda'}{F_\lambda} = \frac{1}{4} = \frac{F_\nu'}{F_\nu}$ (across all wavelengths / frequencies)

No change in S.B
Obj. becomes dimmer but also appears smaller - effects cancel out!

$\Delta\mu = \Delta m + 2.5\log\left(\frac{\Omega_2}{\Omega_1}\right) = -2.5\log\left(\frac{1}{4}\right) + 2.5\log\left(\frac{d_1^2}{d_2^2}\right) = \boxed{0}$

$\frac{I'}{I} = 1$ $\frac{I_\lambda'}{I_\lambda} = 1 = \frac{I_\nu'}{I_\nu}$

$\Delta m = -2.5\log_{10}\left(\frac{1}{4}\right)$

Can't specify $\frac{m'}{m}$! (need to know m!)

2. LARGER MAGNITUDE VALUES = LESS BRIGHT. That's all, no question. Just a reminder.

3. What is the corresponding change in flux if an object's absolute magnitude increases or decreases by 2.5? By 5? By 7.5? 10? You should start to notice a pattern here: *adding* values in logarithmic (read: magnitude) space is equivalent to *multiplying* the values in linear (read: flux) space.

$\boxed{\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \frac{1}{10^4}}$

4. The *luminosity function* of galaxies describes the probability of a galaxy having a certain luminosity L[2]. It is generally written as follows:

$$\phi(L) = \phi^* \left(\frac{L}{L_*}\right)^\alpha e^{-L/L_*}, \qquad (1)$$

where ϕ^*, L_*, and α are just constants. Rewrite the equivalent form of $\phi(L)$, but using (absolute) magnitudes M, i.e. determine $\phi(M)$. *Hint:* $\phi(M) \neq \phi(M(L))$ (just look at the units). But the number of galaxies in a small interval must be the same, i.e. $\phi(M)dM = \phi(L)dL$ must be true[3].

For convenience, let $x \equiv \frac{L}{L_*}$

$\phi(M)\,dM = \phi\left(\frac{L}{L_*}\right)dL \rightarrow \phi(M) = \phi(x)\frac{dx}{dM}$ ①

$M - M_* = -2.5\log_{10}\left(\frac{L}{L_*}\right) \rightarrow x = 10^{-0.4(M-M_*)}$ ②

$\frac{dx}{dM} = 0.4\ln(10)\, 10^{0.4(M_*-M)}$ ③

Plug ② & ③ into ①:

$$\boxed{\phi(M) = \phi^* \, 0.4\ln(10)\, 10^{0.4(M_*-M)(\alpha+1)} \exp\left[-10^{0.4(M_*-M)}\right]}$$

5. An object has a spectrum $L_\lambda = k\lambda^2$ erg/s/Angstrom, where k is just a constant to get the units right. Calculate:

[1] Warning: to make things even more confusing, physicists sometimes use "intensity" for what astronomers call "flux".
[2] Similar to the mass function that we described in class on day 1.
[3] This sort of trick is common throughout physics, whenever we have density functions that can be described using any of a number of nearly interchangeable quantities. In the case of problem (4), $\phi(M)dM = \phi(L)dL$ is just a statement of conservation of number of galaxies. A similar trick can be applied to luminosity/flux densities, i.e. $F_\nu d\nu = F_\lambda d\lambda$. In this case, it is just a statement of conservation of *energy*.

(a) Its total power output over the range 100 to 1000 Angstroms.

$$L_{tot} = \int_{100 Å}^{1000 Å} L_\lambda d\lambda = \int_{10^2}^{10^3} k\lambda^2 d\lambda = \frac{k\lambda^3}{3}\Big|_{100 Å}^{1000 Å} = \boxed{\frac{k}{3}\left((1000)^3 - (100)^3\right)}$$

(b) The shape of the spectrum in frequency space, i.e. determine $L_\nu(\nu)$. If you're stuck, see Footnote 2. Also: plot $L_\nu(\nu)$ and $L_\lambda(\lambda)$.

$$L_\lambda d\lambda = L_\nu d\nu \rightarrow L_\nu = L_\lambda \frac{d\lambda}{d\nu} \quad ①$$

$$c = \lambda\nu \rightarrow \lambda = \frac{c}{\nu} \rightarrow \frac{d\lambda}{d\nu} = \frac{c}{\nu^2} \quad ②$$

$$L_\nu = k\lambda^2 \frac{c}{\nu^2} = \boxed{\frac{kc^3}{\nu^4}}$$

6. Suppose a star cluster has $N = 10^5$ stars. Half of the stars have luminosity $L = L_\odot$ and half of the stars have luminosity $L = 5L_\odot$.

(a) What is the average luminosity $\overline{L}$ of stars in the cluster? What is the absolute magnitude corresponding to $\overline{L}$ (call it $M(\overline{L})$) You can use the fact that the absolute magnitude of the Sun is 4.83?

$$\overline{L} = \frac{(5\times10^4)L_\odot + (5\times10^4)(5L_\odot)}{N} = \boxed{3L_\odot}$$

$$M(\overline{L}) - M_\odot = -2.5\log_{10}\left(\frac{\overline{L}}{L_\odot}\right) \rightarrow \boxed{M(\overline{L}) = 3.6}$$

(b) Now calculate the absolute magnitude corresponding to $L_\odot$ and $5L_\odot$. Using these absolute magnitudes, calculate the average *magnitude* $\overline{M}$ of stars in the cluster.

$$M(L_\odot) - M_\odot = -2.5\log_{10}\left(\frac{L_\odot}{L_\odot}\right) \rightarrow \boxed{M(L_\odot) = 4.83}$$

$$M(5L_\odot) - M_\odot = -2.5\log_{10}\left(\frac{5L_\odot}{L_\odot}\right) \rightarrow \boxed{M(5L_\odot) = 3.08}$$

Average magnitude

$$\overline{M} = \frac{4.83\times(5\cdot10^4) + 3.08\times(5\times10^4)}{N}$$

$$\boxed{= 3.955}$$

(c) Compare $M(\overline{L})$ and $\overline{M}$. Are they the same? Why or why not?

$$\boxed{\begin{aligned}\overline{M} &= 3.955\\ M(\overline{L}) &= 3.6\end{aligned}}$$

Not the same!

The point of this problem was to demonstrate to you that averaging a bunch of numbers in linear space is NOT the same as taking their average in logarithmic space and then undoing the logarithm. That is to say:

$$\bar{x} \neq 10^{\overline{\log_{10} x}}, \qquad (2)$$

where the overlines indicate averages. If we give you a question like this, READ CAREFULLY.

2 Order of magnitude practice

7. **Order of magnitude practice:** I recently flew to New Jersey, from San Francisco. I left at 12pm local time and arrived at 3pm local time. What is the diameter of the Earth? Try not to look anything up.

8. **Order of magnitude practice:** After my last breakup, my friends told me not to worry: "There are plenty of fish in the sea," as the saying goes. Help console me by verifying this and actually calculating how many fish there are in the Earth's oceans. Try to do so without looking up anything. The answer to the last question may be of use. For your own sanity, work only to factors of ten.