

Bohr model & atomic transitions

Late 1800s / Early 1900s, we started taking spectra of hydrogen (& other gases).

Found that if you burn H & ~~observe~~ (observed through) spectrograph, you didn't just see a continuum of light, you instead saw only very specific lines. Why??

* Show picture of H in emission through spectrograph.

Aside: Spectrographs. Basically fancy prisms. Use the fact that ~~accelerated~~ $n = n(\lambda)$ to split light into component wavelengths.

~~Bohr~~ At turn of century (~1900) Rutherford discovered e^- ~~orbit~~ around atomic nuclei. (gold foil experiment).

This motivates Bohr model of atom. Let's pretend e^- in orbit around the nucleus (exactly analogous to star-planet).

Bohr's hypothesis: the angular momentum of the e^- can only take on very specific values.

In day to day life - can take on any infinity of values (walk faster, slower...)

Specifically, Bohr says L comes in "packets" of \hbar . It can have values of $\hbar, 2\hbar, 3\hbar \dots$ but not $1.5\hbar, 7.24\hbar \dots$

Fancy lingo: "quantized"

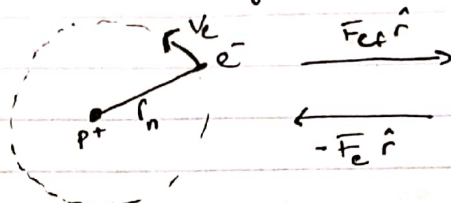
Since L is quantized, it implies E is quantized. ~~But~~ Energy of e^- can only take on very specific values too (we will derive these).

~~Bohr~~ E is quantized.

E quantized explains observations if, as e^- jumps (somehow!) to a lower energy level - that energy has to go somewhere - it goes into a photon! Therefore, only photons of very specific energies ΔE are produced.

btw levels e^- jumped

Now we derive those energy levels E_n , using just the angular momentum quantization.



The energy levels are $E_n = -\frac{e^2}{2r_n}$ (A) (total energy of e^- factor of 2 from v.1)

CGS Unit: I will drop $\frac{1}{4\pi\epsilon_0} \rightarrow 1$ in CGS. E & B have same units ("equal footing" as it should be.)

What is r_n ? Need to use another eqn to go any further.

Use force balance:

$$\frac{e^2}{r_n^2} = \frac{m_e v_e^2}{r_n} \quad (B) \text{ by force balance.}$$

Need expression for v_e too. Use ang. mom:

$$L = n\hbar = m_e v_e r_n \rightarrow v_e = \frac{n\hbar}{m_e r_n} \quad (C)$$

(C) into (B) & solve for r_n :

$$(D) \quad r_n = \frac{n^2 \hbar^2}{m_e e^2} \quad \text{"Bohr radius"} \\ = 0.5 \text{ \AA} / n^2 \quad n=1 \text{ "ground state"}$$

$$a_0 \equiv 0.52 \text{ \AA}$$

Comment: a_0 is a good scale to consider / think about when working with atoms/molecules. "Characteristic length scale"

Now E_n is easy:

$$E_n = -\frac{e^2}{2r_n} = -\frac{e^4 m_e}{2 n^2 \hbar^2} = \frac{-13.6 \text{ eV}}{n^2} \quad \text{Burn into your brain}$$

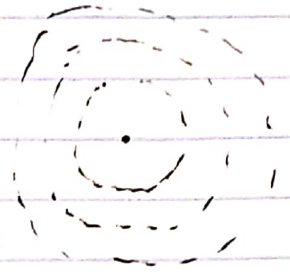
(plug in (D))

We have E_n but we observe $\Delta E \leftrightarrow \lambda$

$$E_f - E_i = -13.6 \text{ eV} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

~~Remember to use the Bohr model to get the energy levels.~~

Comments:



① e^- can go up or down in energy levels.

a) if it goes down it spits out a photon (it lost energy)

b) if it goes up ... how does it go up?

→ has to eat a photon.

→ absorption

→ Must be photon of exactly the right energy difference ΔE .

$\Delta E = \frac{hc}{\lambda}$ to calculate wavelength of emitted/absorbed photon.

② It goes up when hit by a photon ... but why does it go down?
Has anything hit it?

One way to think about it: in nature things prefer to be in lowest possible energy state.

For the aficionados: only jump energy levels when you have a time dependent perturbation to the Hamiltonian (like an EM wave!)
there is a "vacuum field" that produces EM waves briefly (QED subject)

③ $E_n = \frac{1}{n^2}$. So, as $n \uparrow$, $|\Delta E| \downarrow$. We say energy levels are "closely spaced".

SO, λ corresponding to high n transitions is long.

④ A few special transitions:

- a.) Arbitrary $n \rightarrow 2$. "Balmer series" more often!
- $\lambda = 6563 \text{ \AA}$ Super important \rightarrow ~~the~~ $n=3 \rightarrow n=2$ "Balmer- α " or H α
- $n=4 \rightarrow n=2$ H β (because so common)
- b.) Arbitrary $n \rightarrow n=3$ Paschen etc.
- c.) $\rightarrow n=1$ Lyman

⑤ Ionization: ~~Any wavelength~~ corresponds to $n_{\text{fin}} \rightarrow \infty$.

⑥ I derived $Z=1$ case. For HW need to use arbitrary Z . "Bohr-like" $K.E.$