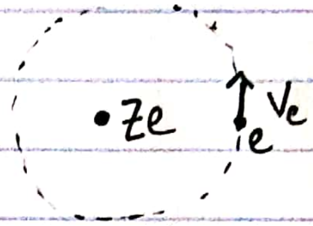
Derivation of Bohr model for arbitrary Z



Energies are $E_n = -\frac{Ze^2}{2r_n}$

$$\left(E = -\frac{q_1 q_2}{2r_n} \text{ with } q_1 = Ze \text{ \& } q_2 = e \right)$$

Use force balance: $\frac{Ze^2}{\cancel{2} r_n^2} = \frac{m_e V_e^2}{r_n}$

Use ang mom for V_e: $L = n\hbar = m_e V_e r_n$

$$\longrightarrow \quad V_e = \frac{n\hbar}{m_e r_n}$$

Plug back into force balance:

$$\frac{Ze^2}{r_n^2} = \frac{m_e}{r_n}\left(\frac{n\hbar}{m_e r_n}\right)^2$$

$$\longrightarrow \quad \boxed{r_n = \frac{n^2\hbar^2}{m_e e^2}\frac{1}{Z} = 0.5\,\text{Å}\,\frac{n^2}{Z}}$$

And back to energies:

$$E_n = -\frac{Ze^2}{2r_n} = \frac{-Ze^2}{2\left(\frac{n^2\hbar^2}{m_e e^2 Z}\right)} \boxed{= \frac{-Z^2 e^2 m_e e^2}{2n^2\hbar^2} = -13.6\,\text{eV}\,\frac{Z^2}{n^2}}$$