1 Gravitational Focusing

In lecture, Eugene derived an expression for the collision rate between two types of particles. In his derivation, we accounted only for physical collisions, ignoring the fact that particles can also “collide” gravitationally. That is, their trajectories can be significantly deflected by gravitational attraction, approximating a physical collision. This is called gravitational focusing. Here we will derive a cross section that accounts for this effect.

a) Consider two identical particles (labeled 1 and 2) of mass $M$ and radius $R$. 1 passes by 2 with impact parameter $b$ and velocity $v_0$. Estimate the change in velocity, $\Delta v$, of 1 as it is deflected from its original trajectory due to the gravitational pull of 2.

b) Gravitational focusing becomes significant when the change in velocity is on order-of-magnitude equal to the initial velocity (i.e. effect on order unity, $\Delta v \approx v_0$). How large does the impact parameter $b$ need to be for this to occur?

c) Using your answer to b), calculate the cross section, $\sigma_{gf}$, for gravitational focusing, in terms of $v_0$, $v_{esc}$, and $\sigma_{old}$, where $v_{esc}$ is the escape velocity of 1’s gravitational field, and $\sigma_{old}$ is the interaction cross section we derived in class, repeated in Equation 1 for convenience.

$$\sigma_{old} = \pi(R_1 + R_2)^2$$  \hspace{1cm} (1)
d) How does $\sigma_{gf}$ compare to $\sigma_{old}$? Do a sanity check: does this relationship make sense, intuitively? Why or why not?

2 Jeans Instability

First, a refresher. Recall that Equation 2 describes the Jean’s length, and whether the radius is greater than or less than this length determines whether an interstellar cloud will collapse.

$$R_J \approx \sqrt{\frac{k_B T}{\mu m_H G \rho}}$$

Jean’s mass is rewritten from the lecture notes in Equation 3.

$$M_J \approx \frac{k_B^{3/2} T^{3/2}}{(G \mu m_H)^{3/2} \rho^{1/2}}$$

You should review your notes to make sure you understand the derivations of these quantities, but I provide them for you here for convenience. You can use $\mu m_H \approx 1.67 \times 10^{-27}$ kg.

a) Concept Check.

(a) As a star collapses, how does its temperature and density change? What does this tell you about how the Jeans radius and Jeans mass change during collapse?

(b) Which two velocities are balanced at Jeans instability? In order for collapse to occur, which one must be greater than the other?

b) Consider the acceleration felt by a particle at the edge of the Jean’s cloud and use a basic kinematic equation relating time, distance, and acceleration to derive the characteristic free-fall time for a Jean’s cloud with mass density $\rho$ to collapse. Note: your answer should depend only on $\rho$ and constants.
c) Typical interstellar molecular clouds have number densities $n_H \approx 10^3 - 10^4 \text{ atoms/cm}^3$ and temperatures of $T \approx 30 \text{ } K$. At what minimum radii ($R_j$) will the clouds collapse? Hint: You will need to rewrite Equation 2 in terms of $n_H$ instead of $\rho$ (use dimensional analysis).

d) What are Jean’s masses (in solar masses) of the clouds described in the part a)?

e) A Jean’s cloud has a number density $n_H = 2 \times 10^3 \text{ cm}^{-3}$. How long will it take the cloud to collapse (ignoring rotation and magnetic fields)? Hint: use your answer from part b).