Physics 7E Practice Midterm
Prof. Hopster
Fall 07

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Problem 1 Figure 1 shows a water tank with a valve at the bottom. If this valve is opened, what is the maximum height attained by the water stream coming out of the right side of the tank? Assume that $h = 10.0$ m, $L = 2.00$ m, and $\theta = 30.0°$, and that the cross-sectional area at A is very large compared with that at B. (15 pts)

Solution See figure P14.54 on page 446 of Serway. You should immediately recognize this as a fluid *dynamics* problem. Your two primary tools for fluid *dynamics* problems are the Bernoulii equation and the continuity equation. We want to use the Bernoulli equation to find the velocity at point B. Once you have the velocity at point B, the problem becomes a simply kinematic motion problem.

The continuity equation tells us that $A_a v_a = A_b v_b$. Rearranging gives $A_b/A_a = v_a/v_b$. Since $A_a \gg A_b$, we have $v_b \gg v_a$. Hence, we can approximate $v_a \approx 0$.

Now use the Bernoulli equation at points A and B. This gives

$$\begin{aligned} \frac{1}{2}\rho v_a^2 + \mathcal{P}_a + \rho g y_a &= \frac{1}{2}\rho v_b^2 + \mathcal{P}_b + \rho g y_b \\ 0 + \mathcal{P}_0 + \rho g h &= \frac{1}{2}\rho v_b^2 + \mathcal{P}_0 + \rho g L \sin\theta \end{aligned}$$

Solving for v_b gives $v_b = \sqrt{2g(h - L\sin\theta)} = 13.28$ m/s.

Now that we have the velocity at B, the maximum height the water exiting the at B attains can be found using $v_f^2 = v_0^2 + 2ay_{max}$. At the maximum height, the y-component of the velocity is zero so $v_f = 0$. v_0 is only the y-component of the velocity at B. Hence, $v_0 = v_b \sin\theta$. Finally, $a = -g$. Hence, $y_{max} = (v_b \sin\theta)^2/(2g)$=2.25 m.

Problem 2 The mass of the deuterium molecule (D_2) is twice that of the hydrogen molecule (H_2). If the vibrational frequency of H_2 is 1.30×10^{14} Hz, what is the vibrational frequency of D_2? Assume that the "spring constant" of attracting forces is the same for the two molecules. (10 pts)

Solution Usually, to find the frequency of oscillation f, you must first find the angular frequency ω and relate the two using $\omega = 2\pi f$. Think of the hydrogen molecule as a mass M_h connected to a spring of spring constant k. Thus, the angular frequency of the hydrogen molecule is given by

$$\omega_h = \sqrt{\frac{k}{M_h}} \tag{1}$$

A similar equation can be written for a deuterium molecule of mass M_d.

$$\omega_d = \sqrt{\frac{k}{M_d}} \tag{2}$$

We are told $M_d = 2M_d$. If we plug this into the equation for ω_d, we get

$$\omega_d = \sqrt{\frac{k}{2M_h}} = \frac{1}{\sqrt{2}}\sqrt{\frac{k}{M_h}} = \frac{1}{\sqrt{2}}\omega_h \tag{3}$$

If we divide both sides by 2π, we get $f_d = f_h/\sqrt{2} = 9.19 \times 10^{13}$ Hz.

Problem 3 A transverse wave on a string is described by the equation

$$y(x,t) = (0.350\,\mathrm{m})\,\sin[(1.25\,\mathrm{rad/m})\,x + (99.6\,\mathrm{rad/s})\,t]. \tag{4}$$

(a) What is the wavelength λ of this wave? (5 pts)

(b) What is the frequency f of this wave? (5 pts)

(c) What is the speed of propagation of the wave? (2 pts)

(d) Now consider the element of string at $x = 0$. What is the time interval between the first two instants when this element has a position of $y = 0.175$ m? (15 pts)

(e) What distance does the wave travel during this time interval? (3 pts)

Solution The form of this wave equation matches our general equation

$$y(x,t) = A\sin(kx - \omega t). \tag{5}$$

Hence, we can immediately read off $A = 0.35$ m, $k = 1.25$ m^{-1}, and $\omega = 99.6$ rad/s. The wavelength is related to the wavenumber by $\lambda = 2\pi/k = 5.03$ m. The frequency is related to the angular frequency by $f = \omega/2\pi = 15.85$ Hz. The velocity of the wave can be found directly from ω and k by $v = \omega/k = 80$ m/s.

If we plug $x = 0$ into the wave equation we get

$$y(0,t) = (0.35)\sin(99.6t) \tag{6}$$

This says that the y position of the string elements at $x = 0$ oscillates between ± 0.35 m. We want to know the first two times it passes through $y = 0.175$. Thus, we want to know when $\sin(99.6t) = 0.5$. $\sin\theta = 0.5$, occurs first at $\theta = 30°$ and then at $\theta = 150°$. Hence,

$$t_1 = \frac{\pi}{6} \times \frac{1}{99.6} = 0.005257\,\mathrm{s}, \qquad t_2 = \frac{5\pi}{6} \times \frac{1}{99.6} = 0.02629, \qquad t_2 - t_1 = 0.021\,\mathrm{s}. \tag{7}$$

Finally, the distance the wave travels during this time is, $d = vt = 1.68$ m.

Problem 4 A pendulum of length L and mass M has a spring of force constant k connected to it at a distance of h below its point of suspension. Find the frequency of vibration of the system for small values of the amplitude (small θ). Assume the vertical suspension of length L is rigid, but ignore its mass. (20 pts)

Solution We can find ω from the equation of motion for the system and find the frequency by $f = \omega/2\pi$. We use Newton's second law for rotations to find the equation of motion

$$\tau_{net} = I\frac{d^2\theta}{dt^2} \tag{8}$$

We want an equation that looks like

$$-\kappa\theta = I\frac{d^2\theta}{dt^2} \tag{9}$$

and then ω is given by $\omega = \sqrt{\kappa/I}$. The I is the rotational inertia, which in this case is a single mass M a distance L away from the pivot. Thus, $I = ML^2$.

When, the pendulum is at an angle θ, there are two restoring torques, one from gravity and one from the spring. The torque from gravity is $\tau_g = -r_\perp F_g = -L\sin\theta Mg$. The negative sign accounts for the fact that the torque from gravity wants to decrease the magnitude of θ. The torque from the spring comes about because the spring is being stretched and compressed by the movement of the pendulum. It is given by $\tau_s = -r_\perp F_s = -h\cos\theta k\, h\sin\theta$. Hence, the net torque is equal to $\tau_{net} = -(MgL\sin\theta + kh^2\cos\theta\sin\theta) \approx= -(MgL + kh^2)\theta$, where we have used the small angle approximation $\sin\theta \approx \theta$ and $\cos\theta \approx 1$, when θ is very small in radians. Pluggin this into equation (8), we get

$$-(MgL + kh^2)\theta = I\frac{d^2\theta}{dt^2}. \tag{10}$$

Comparing this to equation (9), we get that $\kappa = MgL + kh^2$. Plugging this result and $I = ML^2$ into our formula for ω gives

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{MgL + kh^2}{ML^2}} \tag{11}$$

Problem 5 A rope of total mass m and length L is suspended vertically.

(a) If mass is uniformly distributed in the rope, what is the linear mass density μ of the rope? (5 pts)

(b) Find the speed of propagation for transverse waves in this string as a function height y. Hint: The tension increases as a function of height. (10 pts)

(c) Find the time interval it takes for a transverse wave pulse to travel from the bottom of the string to the top of the string. (10 pts)

Solution The linear mass density when the mass is uniformly distributed is just the total mass divided by the total length. Hence, $\mu = m/L$.

The speed of propagation for transverse waves on a string is given by $v = \sqrt{T/\mu}$. The linear mass density is constant so v can only vary with position if the tension varies with position. Since each string element is in equilibrium and not acceleration, the tension at each string element is balanced by the weight of string below it. Hence, the tension increases towards the top of the string because it is supporting more string and is given by $T(y) = \mu g y$. Hence, the velocity as a function of height is

$$v(y) = \sqrt{gy} \tag{12}$$

The instantaneous velocity is given by $v = dy/dt$. Rearranging,

$$dt = \frac{dy}{v} = \frac{dy}{\sqrt{gy}} \tag{13}$$

Thus, to find the time interval it takes the pulse to reach the top, we integrate this equation. At time $t = 0$, the y position of the pulse is zero. After Δt, the position of the pulse is $y = L$.

$$\Delta t = \int_0^{\Delta t} dt = \int_0^L \frac{dy}{\sqrt{gy}} = \frac{2}{\sqrt{g}} \sqrt{y}\Big|_0^L = 2\sqrt{\frac{L}{g}} \tag{14}$$

Problem 6 (Extra credit) A wooden sphere has a diameter of 1.20 cm. It floats in water with 0.400 cm of its diameter above water. Determine the density of the sphere. (25 pts)