Low Light Signal Detection and Intrinsic Noise Sources

Christopher Trinh
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Lab Partner: Scott Woody

ABSTRACT

In this experiment, the power output of an LED is measured from across the room in the presence of external noise sources much greater than the LED such as the room lights and sunlight as well as internal noise sources such as Johnson noise and shot noise. We discuss how such a measurement is possible using phase-sensitive detection with a chopper-wheel and lock-in amplifier.

1. Introduction

Experiments in physics frequently require the measurement of a physical quantity that is very tiny compared to the other signals that are picked up by the detector. One very prominent example is in astronomy where the light emitted by stars and galaxies from far away is measured. Any good astronomer knows that the best measurements must be done far from the city lights on a night when the moon is not out. But even on the darkest night, there is a limit to the faintest object that astronomers can detect. The photons from these objects still reach Earth, but the signal is so small that it will be masked by the faint glow of the sky or the intrinsic noise of the electronics of the telescope.

There are countless other examples in science where low signal detection is necessary and where noise reduction is of the utmost importance. How can low signal detection be accomplished? Many noise sources occur at a particular frequency or are DC in nature. A lock-in amplifier is a piece of equipment that can amplify a signal in a very narrow frequency range. A chopper-wheel can be used to create a periodic signal at a known frequency from the original low signal. A chopper-wheel and lock-in amplifier can be used together to detect a low signal by amplifying the signal at the chopper frequency and attenuating all other frequencies. This is how we will measure the signal for an LED from across the room in the presence of the room lights and sunlight.

2. Equipment

The equipment is the heart of this experiment. The individual pieces will be discussed in considerable depth. A block diagram of the experiment setup is included in the Appendix. The LED signal will be sent through a chopper-wheel. This signal will be detected by a 10-DP Pin Diode.

The current produced will be passed through an SR570 Low-Noise Current Preamplifier. The output signal is then sent to an SR760 FFT Spectrum Analyzer and an SR830 Lock-In Amplifier.

2.1. SR760 - FFT Spectrum Analyzer

The main theorem of Fourier analysis says that any signal can be represented as the sum of sinusoids of different frequencies and amplitudes. A signal can be viewed in either the time domain or in the frequency domain. Viewing a signal in the frequency domain is useful for noise analysis because many noise sources have a predominant frequency component. This can be clearly seen in the frequency domain and aids in the removal of noise.

The mathematical technique for computing the frequency spectrum of a time varying signal is called the Fourier transform. The SR760 computes the Fourier transform of a signal by using an algorithm known as the Fast Fourier transform (FFT). The signal is first digitized by the SR760, which samples the incoming waveform at a fixed sample frequency of 256 kHz for a total of 1024 sample points in the time domain. This yields a total of 512 data points in the frequency domain. There are half as many frequency points due to the fact that the sample frequency must be at least twice as large as the highest frequency component of the incoming signal. The highest frequency in the resulting spectrum is determined by the period of two time samples, 128 kHz in this case. The lowest frequency is the period of the entire time record or 250 Hz in this case. The resolution of the spectrum is equal to the lowest frequency, which is the width of each frequency bin.

The resolution of the SR760 can be adjusted by changing the frequency span. Changing the frequency span effectively reduces the sample rate by digitally filtering the incoming data to limit the bandwidth. A smaller sample rate leads to a longer time record and a longer time record leads to smaller frequency bins and better frequency resolution.

To gain familiarity with the SR760, we experiment with a 0.2 V_{pp}, 400 Hz sine wave. The starting frequency is set at 0 Hz and the frequency span is set to 1.56 kHz. The FFT returns a large peak at 398.438 Hz with an amplitude of -20.44 dBV. It is not surprising that the amplitude measurement is slightly off since there is intrinsic noise from both the SR760 and the function generator.

The discrepancy between the input frequency and the output of the SR760 is a little more interesting. The SR760 has a limited resolution, which determines the values of the frequency bins. Unless there is a bin at exactly 400 Hz, the 400 Hz component of the incoming wave with be placed in the bin closest to 400 Hz. This is the most probable explanation for the SR760 returning 398.438 Hz as the frequency of the 400 Hz sine wave.

The spectrum also shows a large peak at 60/120 Hz. This is a prominent noise source for power lines. They are easily recognizable and should be avoided when possible. There may also be other harmonic noise peaks from the function generator. There is a small peak at approximately

800 Hz. If there are higher harmonics, then the functions is probably not producing a perfect sine wave. This slightly imperfect sine wave would have the perfect sine wave of the correct frequency as the first term in its Fourier series. However, there will also be other harmonics of higher frequency. On the other hand, if no harmonics are visible, it still does not mean that the function generator is producing a perfect sine wave. The amplitude of the higher harmonics may be smaller compared to the other signals and will not be readily visible.

2.2. SR830 - DSP Lock-In Amplifier

The FFT spectrum analyzer will reveal the frequency of the different noise components. Some of these occur at a fixed frequency and can easily be filtered out. However, there are some noises sources, which will be discussed more later, that occur that all frequencies. If the signal of interest is very low compared to this noise, then the signal cannot be recovered. The SR830 Lock-In Amplifier is able to "lock-in" to a signal at a particular reference frequency and attenuate all other frequencies.

The technique used by the lock-in is known as phase-sensitive detection (PSD). The signal coming into the lock-in is first excited at a fixed frequency. The SR830 produces a sine wave at a fixed reference frequency. There may be a relative phase difference between the input signal and the reference signal. If the signal frequency and reference frequency match, the output of the PSD multiplier will be a signal that is DC unless the relative phase difference between the incoming signal and the reference signal is changing in time. This is the phase-sensitive nature of the process.

Two PSD stages can be used to eliminate the dependence on the relative phase between the input and reference signal, thereby producing a DC output from PSD when the input frequency and reference frequency match. All other signals with a frequency different from the reference frequency, such as noise, will produce AC signals that can be removed with a low-pass filter.

To familiarize ourselves with the SR830, we perform some small experiments. First, a 2 V_{pp}, 1 kHz sine wave is fed into the SR830. The sensitivity is set greater than 0.71 V_{RMS} since this is the RMS voltage corresponding to a 2 V_{pp} signal. Next, the time constant of the SR830 must be set. Lets discussed what the time constant of the SR830 is.

The low-pass filter in the SR830 is a simple RC low-pass filter. This type of filter has associated with it a time constant τ, which determines the bandwidth of the filter by setting a cut-off frequency by the relation

$$\tau = \frac{1}{2\pi f}. \tag{1}$$

This relation implies that a large time constant corresponds to a low cut-off frequency and therefore a small frequency bandwidth. As will be shown later, the amount of Johnson noise is a function of the square root of the frequency bandwidth. If the signal being measured has a constant amplitude, i.e. it is DC, a large time constant will produce a more stable signal because the small bandwidth will reduce the amount of Johnson noise present. If instead, the signal being measured had a time-varying amplitude, maximizing the time constant would not be ideal since it takes five time constants for the signal to settle to 99% of its actual value.

The value of the time constant for this part of the experiment is not crucial so we set it to 300 ms and examine the output of the SR830. The output signal is 0.723 V and we expect a voltage of 0.707 V. These two measurements are off by a few millivolts. This is most likely due to the 50Ω terminator resistor. The terminating resistor acts like a voltage divider and a larger terminating resistance will produce a larger input voltage at the SR830.

Next a 2 V_{pp} square wave is used as the input. In this case, the measured output RMS voltage is 0.9254 V. A 2 V_{pp} square of frequency ω can be written as

$$1.273 \sin \omega t + 0.4244 \sin 3\omega t + \cdots \tag{2}$$

The lock-in will measure the $1.273 \sin \omega t$ component of the square wave. Thus, the expected amplitude is $1.273\text{V}/\sqrt{2} = 0.9001$ V. Again, the two measurements are off by a small amount and we again attribute this to the terminating resistor.

There is one last experiment we will perform with the SR830 before moving on. Here, we will examine how fast the lock-in can respond when we change the amplitude of the input signal. Recall that it takes five time constants for the signal to settle to 99% of its actual value. We will use the Labview program LowLight SR830 Lock-In Interface to record the output of the SR830. The time constant is set to 1s, the sample rate to 32 Hz, and the input voltage to 2 V_{pp}. We start taking data and quickly change the input voltage to 0.2 V_{pp}. The data recorded has a characteristic RC decay shape. The horizontal axis of the Labview program is changed into time units from a bin number by using the fact that each bin represents an interval of time equal to the reciprocal of the sample rate. We pick two (t, y) pairs and compute the time constant using the formula

$$\tau = \frac{t_2 - t_1}{\ln(y_1/y_2)}. \tag{3}$$

For a roll-off slope of 6 dB/oct, the time constant is 0.992 s. We can change the roll-off slope of the SR830. This amount to using multiple single RC low-pass filters. The time constant for the single RC low-pass filter is replaced by the effective time constant of the multiple filters. The effective time constant is measured for each roll-off slope and the results are shown in Table 1.

The effective time constant increases as function of the roll-off slope. A steeper roll-off corresponds to sharper attenuation of frequencies above the cut-off frequency. There will be less noise when the roll-off slope is steeper. Thus, the effective noise bandwidth (ENBW) is smaller for steeper roll-off values. It has already been seen that the time constant and frequency bandwidth are related for a single RC low-pass filter by Eq. (3). Table 2 gives the relationship between the effective time constant and the ENBW for each roll-off slope.

The amplitude of the incoming wave can also be changed multiple times during a given run of data. If the SR830 could respond very quickly to these changes, the output would resemble

Roll-off Slope (dB/oct)	Effective Time Constant (s)
6	0.992
12	1.941
18	2.233
24	3.16

Table 1: The effective time constant of the SR830 as a function of the roll-off slope when the time constant of the SR830 is set at 1 s.

Roll-off Slope (dB/oct)	ENBW
6	$1/4\tau$
12	$1/8\tau$
18	$3/32\tau$
24	$5/64\tau$

Table 2: The relationship between the effective noise bandwidth and the effective time constant for each roll-off slope value for the SR830.

a square wave. However, since it takes five time constants for the SR830 to respond, the output resembles the charging and discharging of a capacitor.

2.3. SR760 - Low-Noise Current Preamplifier

The SR760 converts an input current signal into an output voltage signal that is proportional to the input current. This will be used to convert the signal from the photodiode. Using the SR760 is important because current amplifiers have much better amplitude and phase accuracy in the presence of stray capacitance than voltage amplifiers. It is necessary despite that fact that it adds a certain amount of noise to the input signal.

3. Noise Sources

Now that we have discussed the experimental setup, it is time to discuss the sources of noise that will be present when making our measurement. The order in which the noise sources are discussed roughly corresponds to how difficult it is to remove the given noise source, easiest first.

3.1. 1/f Noise

The name of this type of noise comes from its empirical distribution. When f is very small, the amplitude of this noise is quite large. It makes low frequency measurements difficult. However, it is not a limiting factor in this experiment since we have freedom to choose the frequency of the chopper-wheel that excites the LED signal and set the input frequency at the SR830. Thus, $1/f$ noise is not a problem so long as the input frequency at the SR830 is a large value.

The spectrum of $1/f$ noise can be examined using the LowLight FFT Labview program and the SR760. Figure 1 shows the spectrum across the finger of Scott Woody between 0 and 200 Hz. The spectrum shows a peak at 60 Hz, which comes from line noise and that $1/f$ noise is dominant at frequencies lower than approximately 50 Hz.

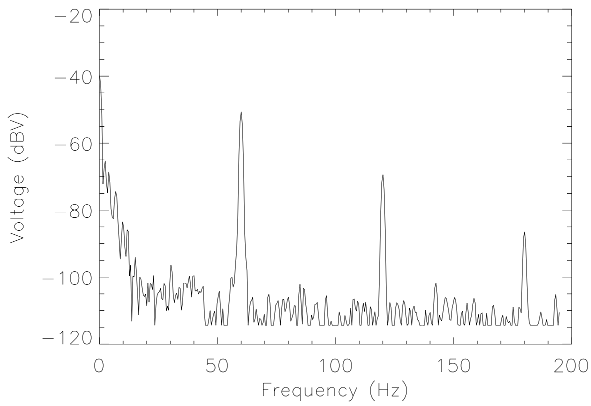

Fig. 1.— The spectrum of $1/f$ noise across the finger of Scott Woody from 0 to 200 Hz.

3.2. Capacitative Coupling Noise

Noise may be introduced into a signal from a nearby AC signal of frequency f and amplitude V through stray capacitance C. The amount of noise current caused by a stray capacitance given by

$$I = 2\pi f\, C\, V. \tag{4}$$

The most problematic source of noise from stray capacitance is power lines with $f = 60$ Hz and $V = 120\ \mathrm{V_{RMS}}$. The stray capacitance from the power lines necessary to induce 1 $\mathrm{mV_{RMS}}$ of noise across a 1 MΩ resistor is

$$C = \frac{V_{\rm noise}}{2\pi f V_{\rm line} R} = 0.022\,\mathrm{pF} \tag{5}$$

This is a fairly small capacitance compared the 33 pF capacitance of a one foot RG/58 coaxial cable.

A coaxial cable is connected to INPUT A of the SR760 and left lying on the table. The spectrum is shown in Figure 2. In the spectrum, $1/f$ noise can clearly be seen towards the left end. There are large noise peaks at approximately 490 Hz, 980 Hz, and 1470 Hz which are most likely harmonics.

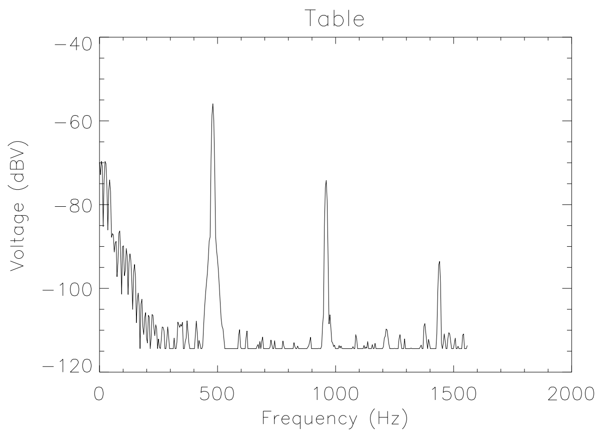

Fig. 2.— The spectrum of capacitative noise from a coaxial cable lying uncoiled on a table.

Next, the span is decreased to 97.5 Hz to examine the 60 Hz noise peak. First, the spectrum is taken with the cable lying flat on the table. Next, the cable is stuff into a cylindrical brass tube before the spectrum is taken. The two are shown in Figure 3. The amplitude of the 60 Hz peak is much smaller when the cable is placed in the brass cylinder. The peak is also smaller if the cable is left dangling off the table or if the cable is coiled up. When the cable is dangling, the capacitative coupling noise is reduce because it is further away from the power outlets. When the cable is coiled, the outer layers shield the inner layers. The brass cylinder reduces capacitive noise by block out external static electric fields like a a Faraday cage.

3.3. Microphonic Noise

Mechanical noise like the oscillation of coaxial cables can manifest itself as electrical noise. Microphonic noise may not be a major problem since it can be minimized by reducing mechanical vibrations near the experiment as much as possible, constraining cable that carry sensitive signals, and using low noise cables that are designed specifically to combat microphonic noise. Nevertheless, lets estimate the magnitude of microphonic noise caused by shaking one meter of RG/58 coaxial cable at frequency of 10 Hz. To do this three assumptions are made. First, the cable has 1 V across it and a capacitance of 33 pF/foot. Next, the signal goes into a scope with a 1 MΩ input impedance. Finally, the shaking causes a change in capacitance $\delta C = 1$ pF. The total capacitance of the cable is $C = 108$ pF The differential equation governing the cable is given by the time

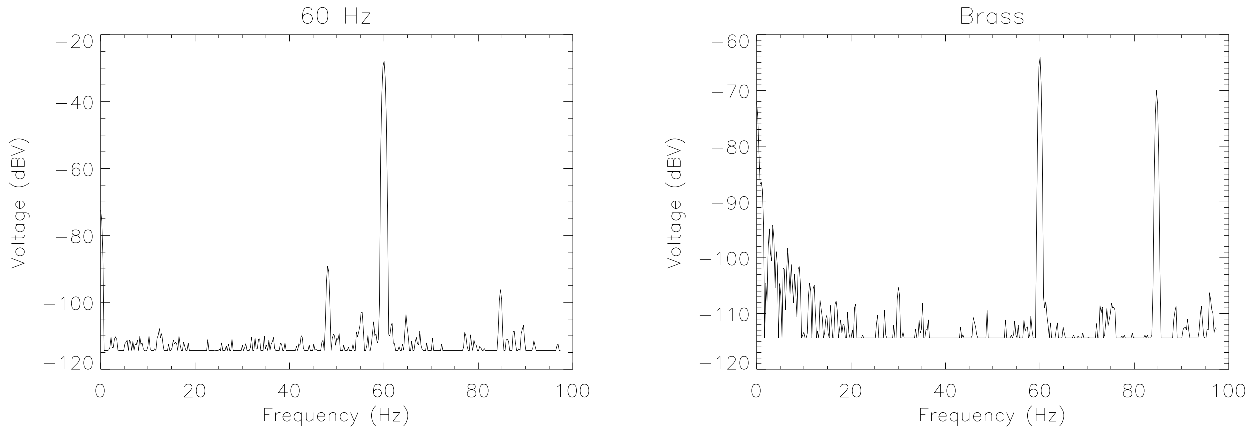

Fig. 3.— The spectrum for capacitative coupling noise in a coaxial cable when unshielded and shielded by a brass cylinder.

derivative of $Q = CV$, which is

$$C\,\frac{dV}{dt} + V\,\frac{dC}{dt} = I. \tag{6}$$

The shaking does not change the voltage across the cable so $dV/dt = 0$. The induced current is equal to

$$I = \delta C \cdot V \cdot f = 10^{-8}\,\mathrm{A} \tag{7}$$

which corresponds to a voltage noise of $V = 0.01$ V. Note that the ratio of the induce capacitance to the capacitance is equal to 1/100 and this is equal to the ratio of the induced voltage to the original voltage.

The spectrum of microphonic noise is examined in realtime using the SR760. First, the coaxial cable is shaken vertically to resemble a sine wave. Next, the coaxial cable is spun around a horizontal axis like a jump rope. In both cases, multiple peaks slowly arise at regular intervals. The peaks correspond to different harmonics and the fundamental that corresponds to the frequency of spinning or shaking. The peaks are broadened because the frequency of shaking and spinning are not well fixed at a constant rate.

3.4. Shot Noise

Shot noise arises because of the discrete nature of electrical charge. This quantization of charge leads to noise in the flow of electric charge or current due the non-uniformity of the flow and is present at all frequencies. The RMS current noise due to shot noise is given by

$$I = \sqrt{2eI_0\,\Delta f} \tag{8}$$

where e is the charge of the electron, I_0 is the RMS current signal, and Δf is the frequency bandwidth. Note that a stronger current signal produces more shot noise! If the current signal is large, shot noise may not be a problem since it only grows as the square root of the current signal. However, if the current signal is small, then the magnitude of shot current will have a comparable amplitude making measurements much more difficult. The shot noise is equal to the current when $I_0 = 2e\,\Delta f$. Shot noise is different than the other noise source discuss thus far in that the noise comes from the signal of interest itself, which adds to the difficulty in minimizing it.

Lets compute the shot noise for the 10-DP Pin Diode. The output current of 10-DP Pin Diode will be around 10 μA. The frequency bandwidth for the SR830 is equal to the ENBW. For a roll-off slope of 24 dB/oct and a time constant of 30s, Table 2 gives $\Delta f = 0.0026$ Hz. Thus, the RMS shot noise is $I = 9.12 \times 10^{-14}$ A/$\sqrt{\text{Hz}}$

3.5. Johnson Noise

Another noise source that is present in all electronic systems at all frequencies is the noise produced by random thermal fluctuations of electron density in every resistor. This noise is known as Johnson noise and is manifest as a RMS voltage given by

$$V = \sqrt{4k_bTR\Delta f} \tag{9}$$

where k_b is the Boltzmann constant, T is the temperature, R is the resistance, and Δf is the frequency bandwidth. Johnson noise can also manifest itself as a RMS current by

$$I = \frac{V^2}{R^2_{\text{source}}}. \tag{10}$$

We calculate the current noise for the Pin-10DP diode that will be use to detect the light signal from the LED. For this, we need the source impedance of the SR830, which is 2 MΩ. Again, the frequency bandwidth is taken to be the ENBW of the SR830. Using the same roll-off slope and time constant as in the shot noise exercise gives $\Delta f = 0.0026$ Hz. Thus, the Johnson noise for the photodiode detector is $V = 2.51 \times 10^{-8}$ V/$\sqrt{\text{Hz}}$. Johnson noise would be reduced if a current signal came for a high impedance source because of the R^2_{source} term in the denominator of Eq. (10). For a voltage source, a low impedance source would be preferred because of the $R^{1/2}$ dependence shown in Eq. (9).

We will now test Eq. (9) for the SR830. Using a pair of coaxial cables, a pair of clips with a 100k resistor is connected to INPUT A/I of the SR830. The internal reference of the lock-in is used. The reference frequency is set at 1 kHz to avoid $1/f$ noise and 60/120 Hz line noise and its harmonics. A roll-off slope of 24 dB/oct is used. The time constant is set to 30 ms and seven runs of data are taken. For each data run, the average signal and the standard deviation of the signal is computed. It is the standard deviation of the signal that is the average magnitude of Johnson noise. The process is repeated for different time constants. The results are listed in Table 3. The measured values are off by factor of $\sqrt{2}$.

The SR830 measures the standard deviation of RMS voltage values. Thus, it is also an RMS voltage. The magnitude of Johnson noise given by Eq. (9) is not. The RMS in the equation refers to the fact that Johnson noise is random because of the random nature of the thermal fluctuations. The expected value of Johnson noise is thus zero. Thus, to find the magnitude, one would need to find the mean squared deviation from zero. If this is done for a distribution of peak voltages, then the RMS value is a peak voltage. Thus, Eq. (9) gives an RMS value of peak voltage whereas the SR830 measures the RMS value of RMS voltages. Hence, the two are off by a factor of $\sqrt{2}$. A column for the total noise is included. The input noise is 5 nV/$\sqrt{\text{Hz}}$. The total noise is then given by $V_{\text{tot}} = \sqrt{V_J^2 + V_{\text{input}}^2}$.

The plot of Johnson noise as function of $\sqrt{\text{ENBW}}$ is shown in Figure 4. There is only one value at $\tau = 1$ that does not agree with theory. This is a long time constant so it takes a long time to take all the data. The cables may have been moved during the data run and add some microphonic noise. This point is not plotted in Figure 4.

τ (s)	Measured V_{rms} (V)	Expected V_{RMS} (V)	ENBW (Hz)	Total Noise
0.03	4.63×10^{-8}	6.56×10^{-8}	1.614	4.66×10^{-8}
0.1	2.88×10^{-8}	3.59×10^{-8}	0.88	2.92×10^{-8}
0.3	1.61×10^{-8}	2.08×10^{-8}	0.51	1.69×10^{-8}
1	2.04×10^{-8}	11.37×10^{-9}	0.28	2.04×10^{-8}
3	8.9×10^{-9}	6.56×10^{-9}	0.16	1.02×10^{-8}

Table 3: The magnitude of Johnson noise across a 100k resistor as measured by the SR830 for different time constant values.

4. Low Light Signal Detection

The relevant noise sources and equipment have been discussed. It is finally time to put everything together and see if the low power output of an LED can be measured from across the room in the presence of the room lights and sunlight and other intrinsic noise sources.

First, we will see if the LED signal can be detected at all. The DP-10 Pin Diode is connected to the SR570 input. Next, the output of the SR570 is connected to the A Input of the SR760 and the A/I Input of the SR830. The internal reference of the SR830 is used. The LED and chopper-wheel are controlled with the remote control box. The span of the SR760 is set to 1.56 kHz. The setting of the equipment are fiddled with until a good signal to noise ratio is achieved and the signal from the LED is dominant over the relevant noise. The frequency of the peak can be shifted by changing the chopper frequency. We check how accurate the frequency given by the

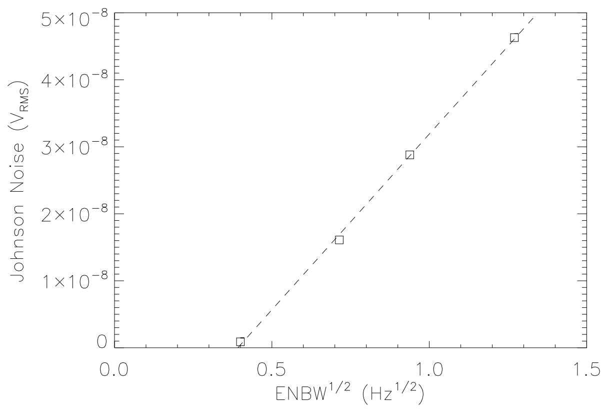

Fig. 4.— The magnitude of Johnson noise as a function of the square root of the ENBW of the SR830.

SR760 and the actually chopper frequency is. The reference signal from the chopper is fed into the REF IN on the SR830. This reveals that the chopper frequency is really 467.2 Hz, whereas the SR760 frequency is 468.75 Hz. This is quite a high level of agreement and the performance of the equipment should be sufficient for the task at hand.

The previous experiment showed that the lock-in can successfully pick up the LED signal and that the signal can be distinguished from the noise. This will be done again more carefully and the output will be recorded this time for analysis. First, an appropriate chopper frequency must be chosen. We chose a frequency of 1.016 Hz. This frequency is large so $1/f$ noise is not a concern. It is not a multiple of 60/120 Hz so line noise is not a problem. To reduce Johnson noise and shot noise, a large time constant ($\tau = 30$ s) and roll-off slope (24 dB/oct) is used. Recall that a large time constant and steep roll-off correspond to small frequency bandwidth and both Johnson noise and shot noise are proportional to the square root of the bandwidth. Finally, microphonic noise is avoid by not touching the coaxial cables during data runs. This is just about all that can be done to minimize noise.

The 1.016 kHz peak is first viewed on the SR760 to see a clear signal is produced. The spectrum is shown in Figure 5. The signal to noise is not fantastic but will get the job done.

Next, the lock-in is used to measure the LED signal. Four data runs are taken each taking about 90 minutes. For each data run, the LowLight SR830 Labview program computes the average signal value and the standard deviation. The four values from each data run are then average, with each run equally weighted. The average measured signal value is 3.78×10^{-5} V with a standard deviation of 1.96×10^{-7} V.

To get a feel for whether the results obtained are reliable, we compared the measured standard

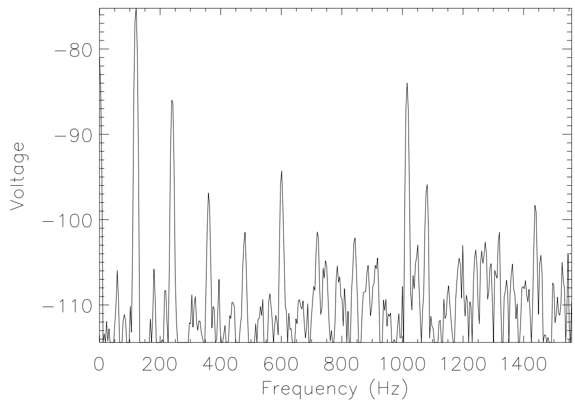

Fig. 5.— A spectrum showing the 1.016 kHz peak that is the LED signal picked up by the 10-DP Pin Diode and sent through the SR760 FFT spectrum analyzer.

deviation, which is a measure of the noise, to the expected Johnson and shot noise, which are the dominant noise sources. The expected magnitude of Johnson and shot noise are 2.51×10^{-8} and 1.826×10^{-7}, respectively. The sum of the two is 2.08×10^{-7}. This is very close to the actual measured value of 1.96×10^{-7} and is the indication of good data.

How can the power output of the LED be computed from signal at the lock-in? First, the 10-DP Pin Diode converts a power into a current from an electron and electron hole pair, which are created by an incoming photon. The conversion for this process is approximately 0.35 A/W. Next, the current passes through the SR570, which changes the current into a voltage according to some sensitivity level, which is 1 μA/V in this experiment. Finally, this voltage is fed into the SR830. The SR830 sees a triangle wave with a peak to peak amplitude equal to the LED power output after passing through the 10-DP Pin Diode and the SR570. The first component of the Fourier series of a 2 $V_{\rm pp}$ triangle wave with frequency ω is

$$f(t) = \frac{8}{\pi^2}\sin\omega t = 0.81\sin\omega t \tag{11}$$

Like the example of the square wave in the discussion of the SR830, this is the component that the SR830 will lock into. Thus, a 1 $\rm V_{peak}$ triangle wave corresponds to a 0.81 $\rm V_{peak}$ sine wave. Since the SR 830 displays RMS voltage, the output is multiplied by $\sqrt{2}$ to give a peak voltage of 5.346×10^{-5} V. Next, it is divided by 0.81 to transform the voltage from the peak voltage of the Fourier decomposed sine wave into the original peak voltage of the triangle wave. This gives 6.60×10^{-5} V. Since the LED power corresponds to the peak to peak voltage, we divide this peak voltage by 2 to give 3.3×10^{-5} V. Now we multiply by the sensitivity of the SR570 to get the current detected by the 10-DP Pin Diode, which is 3.3×10^{-11} A. Finally, multiplying by the 10-DP Pin Diode conversion gives the power detected by the photodiode detector, which is 9.43×10^{-11} W.

The number for the power detected by the 10-DP Pin Diode can be compared to what is theoretically expected to be the power from the LED at a distance of 24 ft. The intensity of the LED is assumed to follow

$$I(r) \propto C\,\frac{\cos^2\theta}{r^2} \tag{12}$$

If the constant C were known then the power at the 10-DP Pin Diode could be approximated as

$$P_{\rm photodiode} \approx I(\theta = 0, 24{\rm ft}) \cdot A \tag{13}$$

where A is the area of the 10-DP Pin Diode. The power at the surface of the photodiode detector is actually the integral of the intensity over the surface area. However, since the area of the photodiode detector is small compared to the distance between the 10-DP Pin Diode and the LED, the intensity is approximately constant over the area of the photodiode detector.

To find C, note that the power emitted by the LED at its surface is the integral of the intensity over the LED surface. This integration yields

$$P = C\,\frac{2\pi}{3} \tag{14}$$

The power emitted by the LED is 1.26 mW at 635 nm. With these ingredients, the power at the 10-DP Pin Diode is given by

$$P_{\rm photodiode} = \frac{3P}{2\pi r^2} \cdot A. \tag{15}$$

The area of the 10-DP Pin Diode is approximately 0.001 m^2 and $r = 24\,{\rm ft} = 731.52$ m. This yields a theoretical power detected by the photodiode detector to be 1.13×10^{-9} W. This is approximately a factor of 10 off from the measured value. The agreement between the two values is quite good considering the coarseness of many approximation and estimates for physical quantities such as the area of the 10-DP Pin Diode and the distance between the photodiode detector and the LED.

5. Conclusion

The lock-in amplifier and chopper-wheel combination is a very powerful tool in physics because of the ubiquity of low level signals. This experiment has provided us with better understanding of phase-sensitive detection and the Fourier transform. Both are extremely useful for measurements in physics and we are certain to encounter them again. We have also seen how cumbersome noise can be. Yet, we have also gain a better appreciation for the techniques used to minimize it. Even more so than the Fourier transform and phase-sensitive detection, there is absolutely no way to escape noise if we choose to continue in physics.

6. Acknowledgements

I would especially like to thanks Scott Woody for being my lab partner (a great one) this semester. I also like to thanks Don Orlando and Daniel McQueen for their help and making the

111 lab an interesting place to be.

7. References

Stanford Research Systems SR570 Low-Noise Current Preamplifier Manual
Stanford Research Systems SR760 FFT Spectrum Analyzer Manual
Standfor Research Systems SR830 DSP Lock-In Amplifier Manual

8. Appendix