An Investigation of Solar Rotation using Spectroscopy

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ABSTRACT

We use the Echelle Spectrograph in the Undergraduate Laboratory at the University of California at Berkeley to conduct a spectroscopic analysis of the Sun. The solar data is calibrated by examining the emission lines from a neon source. We use neon spectra to deduce the spectral resolution and wavelength scale of the spectrograph. From the calibrated solar data we find the doppler shift of the Sun as a function of time. We then deduce the rotational speed of the Sun and use it to compute its radius.

1. Introduction

The astronomer has two key tools available: photometry and spectroscopy. Photometry, the measurement of the quantity of light emitted by stars, has already been the subject of investigation in my previous reports. In this report, we will focus on spectroscopy. The focus of spectroscopy is not on the *total* amount of light but rather the *relative* amount of light emitted by a star at different wavelengths.

The spectra of a star or any light source for that matter is a plot of the flux as a function of wavelength. There are two types of spectra, discrete and continuous. Discrete spectra contain only a few relatively distinct peaks. The spectrum of neon is discrete and is shown in Fig. 1. The Sun on the other hand has a continuous spectrum as shown in Fig. 2. The main difference in discrete versus continuous spectra is the composition of light from the source. Light from the sun contains an infinite number of wavelengths and the spectrum of the sun shows emission all wavelengths whereas the neon spectrum does not emit at all wavelengths.

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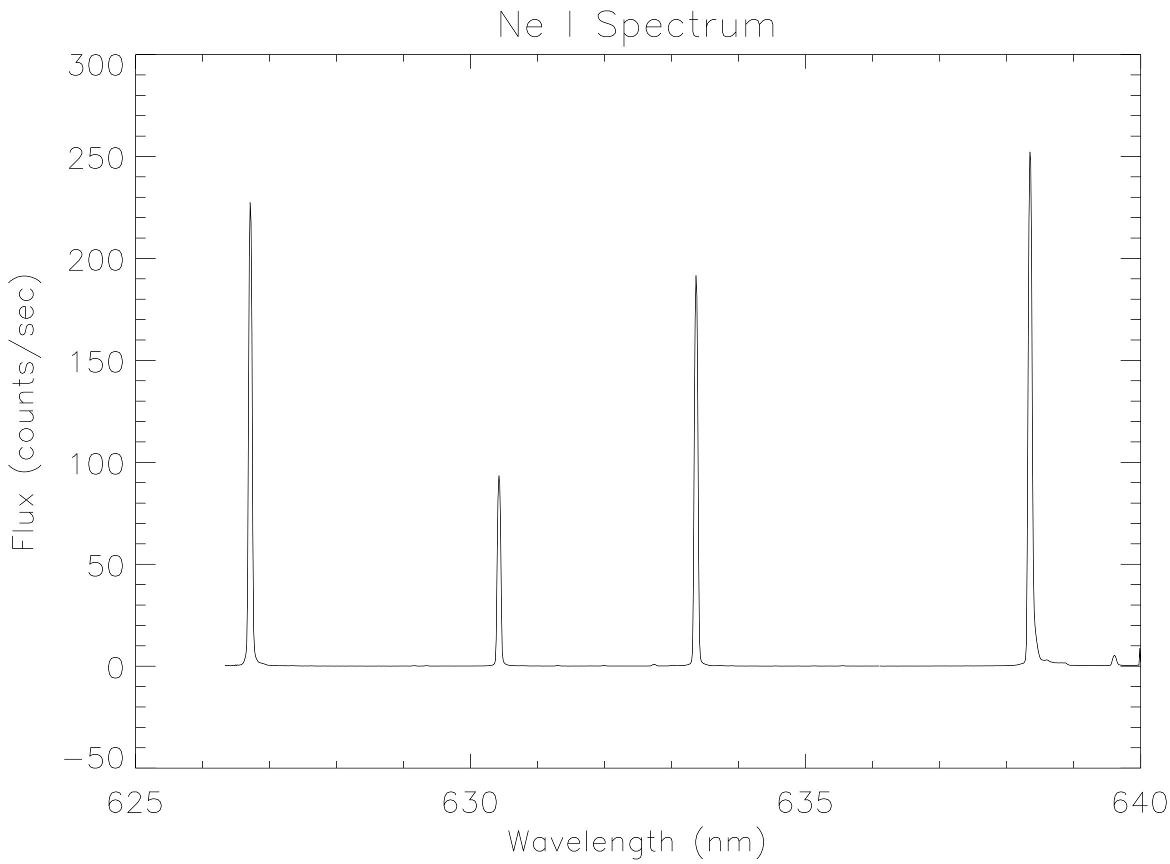

Fig. 1.— A discrete spectrum of neon. The peaks correspond to 626, 630, 633, 638 nm. We can measure the spectral resolution of the spectrograph by finding the FWHM of the peak and dividing by the central wavelength of the peak. We average the four values and get a spectral resolution of 11100 or 27 km/s.

Examining the spectra of the Sun allow us to measure some interesting properties. First, since the Sun is rotating, the light we measure from the Sun will be Doppler shifted. The direction of the Doppler shift depends on whether we image a portion of the Sun that is rotating away from us or towards us. Suppose the Sun's angular momentum vector points directly North in the observers coordinates. If we image the Sun from west to east directly across the equator, the Doppler shift will decrease as a function of position since the western limb is rotating toward us, the center of the Sun is rotating tangentially to our line of sight, and the eastern limb is rotating away from us. The Doppler shift will be reflected in the spectra of the Sun. The peaks in the spectra will be shifted because of the Doppler shift.

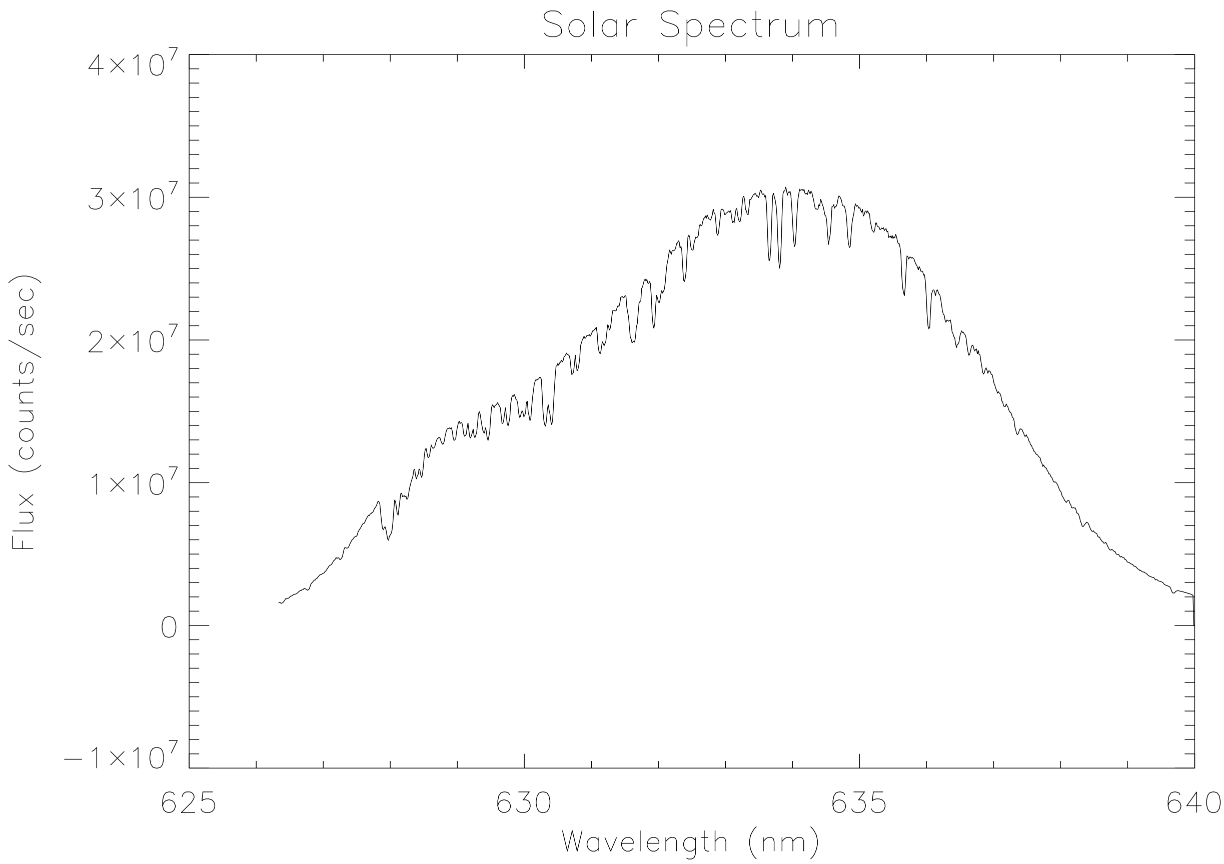

Fig. 2.— A continuous spectrum of the Sun. Light from the Sun is composed of an infinit number of wavelength so its spectrum is continuous.

Since the Earth is rotating, we can let the Sun drift across a telescope connected to the spectrograph and measure the Doppler shift of the Sun as a function of time. After correcting for geometry, we can use the slope of this graph to compute the rotational speed of the Sun. Armed with this and the rotational period of the Sun, we can compute its radius.

2. The Spectrograph

The spectrograph consists of several components. The first is a fiber source with a diameter of 100 μm. This is where the light comes out. Next is a collimator with a focal length of 179 mm. The light emitted from the fiber source diverges radially and the collimator focuses the light so that the incident rays are all parallel. The most essential part of the spectrograph is the diffraction grating. The Echelle spectrograph has a reflection grating with a groove density of 80 mm^{-1}. The light that is reflected by the grating is focused by a camera lens on to a CCD. The camera has a focal length of 200 mm and each pixel element of the CCD is 13 μm. The full angle 2θ between the incident and reflected beams is $10\,^{\circ}$. These properties of the spectrograph as listed in Table. 1.

Spectrograph Property	Value
Groove spacing	0.0125 mm
Blaze angle	64.5°
Fiber diameter	100 μm
f_{col}	180 mm
f_{cam}	200 mm
CCD pixel size	13 μm
θ	5°

Table 1: This table lists the nominal properties of the spectrograph.

We can derive further properties of the spectrograph. Our starting point in the grating equation

$$m\frac{\lambda}{\sigma} = \sin\alpha + \sin\beta \tag{1}$$

where m is the order of interference, α is the angle of incidence, β is the angle of reflection, and σ is the length of the diffracting element. Using a technique known as blazing, the directions of the constructive interference and specular reflection can be made to coincide by choosing the appropriate shape for the periodic structure of the diffracting element. The blaze angle is δ and we have $\alpha = \delta + \theta$ and $\beta = \delta - \theta$. Using this and the trigonometric identity $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ yields

$$m = 2\,\frac{\sigma}{\lambda}\sin\delta\cos\theta \tag{2}$$

for the order on the blaze.

The spectrograph is only effective over a small range of wavelengths called the free spectral range $\Delta\lambda$. The free spectral range is the maximum wavelength range for a given λ that does not contain an emission peak of the next order of interference. To find this, we find $\Delta\lambda$ such that the m peak of λ coincides with the $m+1$ peak.

$$m(\lambda + \Delta\lambda) = (m+1)\lambda \qquad \Leftrightarrow \qquad \Delta\lambda = \frac{\lambda}{m} \tag{3}$$

Thus the maximum and minimum wavelengths around λ are

$$\lambda_{\rm max} = \lambda + \frac{\Delta\lambda}{2}, \qquad \lambda_{\rm min} = \lambda - \frac{\Delta\lambda}{2} \tag{4}$$

From the maximum and minimum wavelengths of the order, we can find the angular spread of the order $\Delta\beta = \beta_{\rm max} - \beta_{\rm min}$. Solving Eqn. 1 in terms of β we have

$$\beta = \sin^{-1}\left[\frac{m\lambda}{\sigma} - \sin\alpha\right]. \tag{5}$$

We use this equation for maximum and minimum wavelengths and taking the difference for the angular spread of the order.

Using basic trigonometry, we find that the length of the CCD is

$$l_{\rm CCD} = 2 f_{\rm cam} \tan\frac{\Delta\beta}{2} \tag{6}$$

We can convert this number to pixels since we know the size of each pixel on the CCD. Thus we can compute the spectral dispersion by

$$\frac{d\lambda}{d{\rm pix}} = \frac{\Delta\lambda}{l_{\rm CCD}} \tag{7}$$

We need the spectral dispersion to compute the wavelength scale of the CCD. The wavelength scale allow us to plot flux versus wavelength rather than flux versus pixel. Also $l_{\rm CCD}$ is the width of the order in pixels. We can also state the width of the order in units of length since we know the size of each pixel. Knowledge of the width of the order allow us to choose our image size appropriately.

The spectrograph is an optical system and the image of the fiber on the CCD may be magnified. If the spectrograph were purely an optical system, i.e. containing only lenses, then the magnification would simply be $M = f_{\rm cam}/f_{\rm col}$. However, because of the dispersion from the diffraction grating, we must include an anamorphic term

$$\frac{\partial\beta}{\partial\alpha} = \frac{\cos\alpha}{\cos\beta} \tag{8}$$

Hence

$$M = \frac{f_{\rm cam}}{f_{\rm col}} \cdot \frac{\cos\alpha}{\cos\beta} \tag{9}$$

The size of the fiber on the CCD in the dispersion direction will be $d_{\rm CCD} = d_{\rm fiber} \cdot M$. Since we know the size of each pixel we can express this number in pixels.

The most important properties of the spectrograph is the spectral resolution. It is a measure of how well the spectrograph separates wavelengths. Our results are fundamentally limited by the spectral resolution. The higher the spectral resolution the better we can distinguish between two wavelengths. We can compute it by

$$R = \frac{\lambda}{\delta\lambda} = \frac{\sin\alpha + \sin\beta}{\cos\alpha}\frac{f_{\rm col}}{d_{\rm fiber}} \tag{10}$$

We can state the spectral resolution in kilometers per second by using the equation for the Doppler shift

$$v = \frac{\delta\lambda}{\lambda}c \tag{11}$$

where c is the speed of light.

We compute these derived properties of the spectrograph for $\lambda = 633$ nm and list them in Table. 2

Spectrograph Property	Value
Order on the blaze	35
Free spectral range of the order	18 nm
Maximum wavelength of the order	642 nm
Minimum wavelength of the order	624 nm
Angular spread of the order	5.3°
Magnification	0.77
Spectral resolution element	5.9 pixels
Spectral dispersion	0.013 nm/pixel
Spectral resolution [R=$\lambda/\delta\lambda$]	9243
Spectral resolution [km/s]	32.5 km/s
Width of order on CCD [mm]	18 μm
Width of order on CCD [pixels]	1385 pixels

Table 2: This table lists the derived properties of the spectrograph.

3. Data Calibration Using a Neon Source

The spectrum of neon is discrete and has characteristic emission peaks. We use this property of neon to find a conversion between wavelength and pixel position. This will allow us to plot flux as a function of wavelength for light emitted by the Sun.

We change the setting so that our images will be 1056 by 40. The spectra are really one-dimensional but we image 40 pixels in the vertical direction since the dispersion direction is not exactly horizontal. We illuminate the spectrograph with a neon source while using the 633 nm filter and take 50 science image and 50 dark images with an exposure time of 0.02 seconds. We also illuminate the spectrograph with a keychain light and take 50 exposures to make a flat field.

The images of the neon source contain four elliptical dots spread out across the CCD as shown in Fig. 3. Each dot corresponds to an image of the fiber source at a different wavelength. They are not circular because of the anamorphic magnification term in the dispersion direction. The dispersion direction is not oriented exactly along the rows of the CCD. To correct for this, we compute the two-dimensional pixel position of each of the four dots in all 50 images. We take the average position through all 50 images for each dot. The best fit line through these four points gives a slope that can be used to compute the angle of rotation.

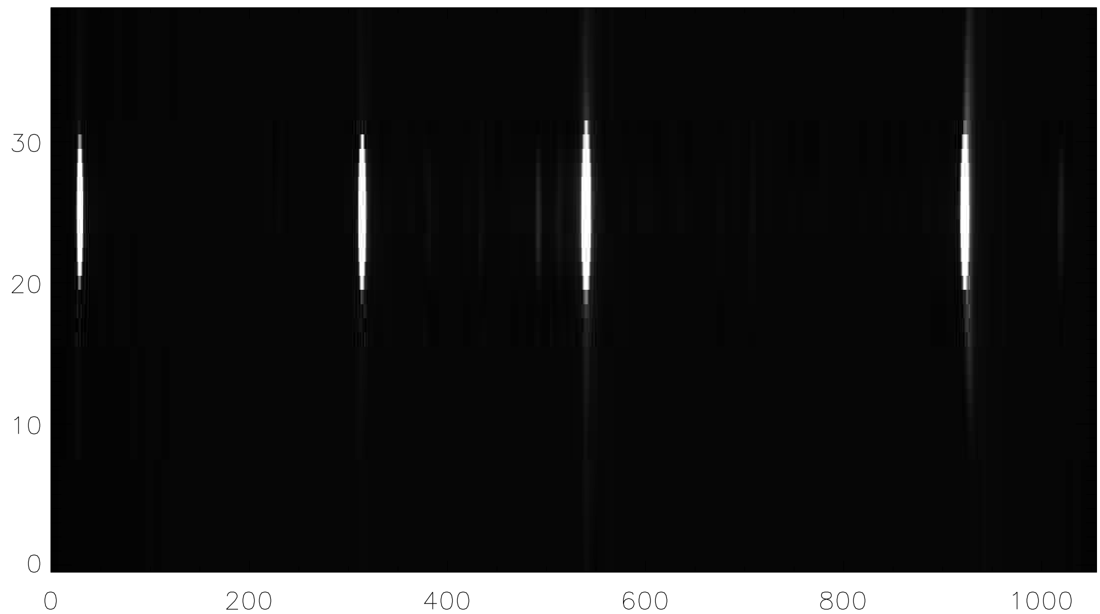

Fig. 3.— A 1056 by 40 image of the neon source. The dispersion of the spectrograph is not aligned exactly along the rows of the CCD. We find the angle of rotation by find the position of the four dots and fitting a line through them. This image has already been corrected so all the dots line up horizontally.

Now that the dispersion direction is oriented exactly along the rows, we simply total each column and divide by the exposure time. We make a dark spectrum and flat spectrum in the usual manner. We correct our neon spectra by subtracting off the dark spectrum and dividing by flat spectrum. At this point, our spectra are in units of flux versus pixel. We average the signal in each pixel through all 50 spectral.

We identify the emission peaks of our neon spectrum using the online Ne atlas at Kitt Peak National Observatory. The lines correspond to 626.6, 630.4, 633.4, and 638.2 nm respectively as shown in Fig. 1. We find the one-dimensional pixel position of each peak by applying a mask and finding the center of light. We plot wavelength versus pixel in Fig. 4. We fit a line to the array of pixel positions and the array of corresponding wavelengths and this yields a wavelength scale or spectral dispersion of $0.013 \pm (1.325\times10^{-5})$ nm/pixel. Now that we have our wavelength scale, we are ready to calibrate our solar data.

Before we move on to the solar data, we can use the neon spectrum to measure the spectral resolution of the spectrograph. The spectral resolution is the lower limit on whether or not we can distinguish the emission peaks of two different wavelengths. If the wavelength difference is too small, the peaks will appear to coincide. For two peaks to be resolvable, the maximums of the peaks must differ by the full width at half maximum (FWHM) of the larger peak. We fit a Gaussian to each emission peak in the neon spectrum and calculate the FWHM by

$$\mathrm{FWHM} = 2\sqrt{2\ln 2}\sigma \tag{12}$$

where σ is now the width of the Gaussian. With the FWHM for each emission peak we calculate the spectral resolution by dividing the peak wavelength by the FWHM for that peak. We take the average of the four values of the spectral resolution and find a final value of 11100. In kilometer per second, the spectral resolution is 27 km/sec. This yields a percent difference of 17/calculated in Table. 2. The discrepancy is likely due to the face that the emission profile of the fiber source is not exactly a top hat profile and more of a gaussian shape.

4. Doppler Shift of the Sun

A total of 42 solar images were taken. Every image is rotated using the angle found from the neon data. The Sun was directly over the fiber in images 6 through 23. The rest of the images contain mostly scattered light. We use images 30 through 42 to make a background sky spectrum and subtract it from each solar spectrum. Finally the wavelength scale found using the neon data is used to plot flux vesus wavelength for the Sun as in Fig. 2.

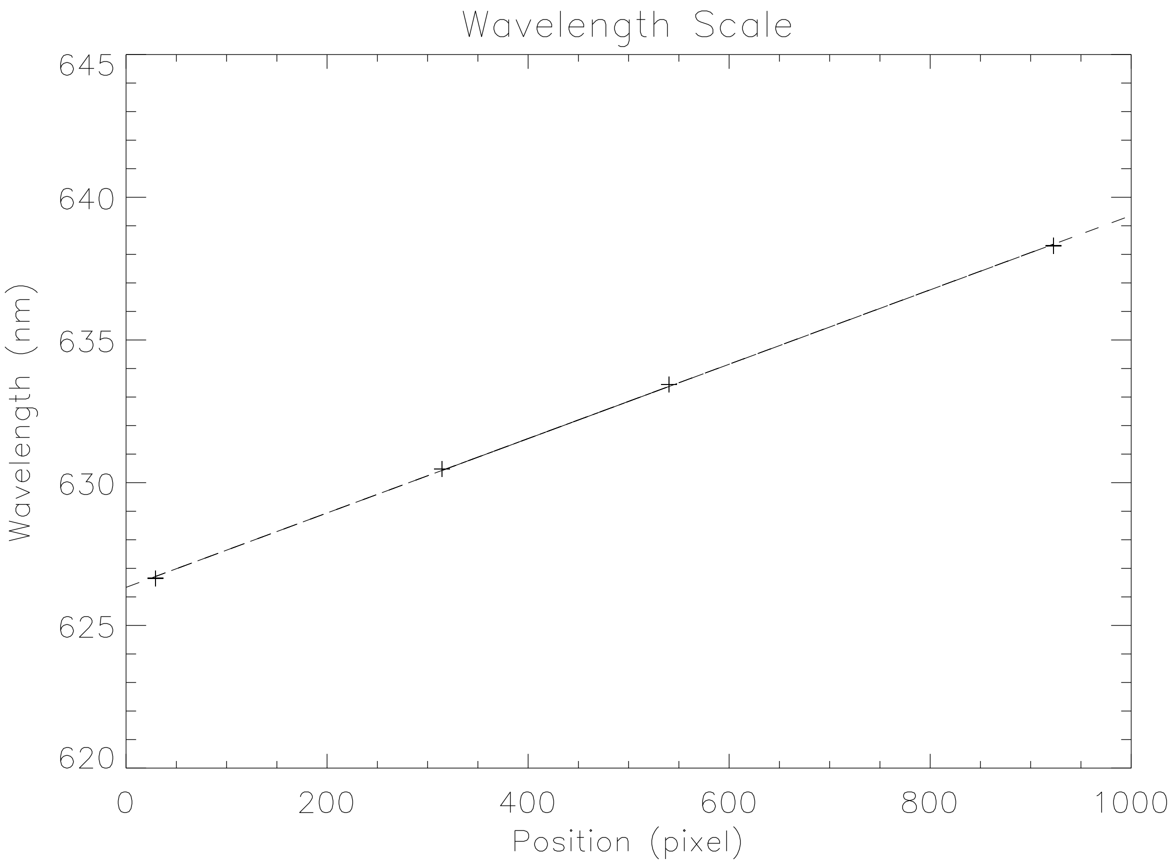

Fig. 4.— A plot of the wavelength of the emission peak as a function of pixel position. The best fit line gives a conversion between pixel position and wavelength so that we can plot flux as a function of wavelength.

We now wish to find the shift of the peaks in the solar spectrum as the Sun moves across the fiber. The movement is much to small for the human eye too perceive when looking at the different solar spectrum as shown in Fig. 2. The technique we will use to measure the wavelength shift from spectrum to spectrum will involve using IDL to correlate the images.

Before we convolve images, we must modify our data slightly. In correlating the images, we hope to find the offset at which the different peaks match up best. Thus, we subtract off a smoothed version of each spectrum. This removes the large scale structure in the solar spectrum that is seen in Fig. 2 and it emphasizes the small scale variations. We also multiply the smoothed image by a Hanning window so that we do not run into trouble when we sending the spectra through Fourier space.

Now we choose the middle image of our 18 solar spectrum and correlate all 18 images with it. With find the pixel position of the peak of the correlation image by masking and finding the centroid. We subtract off the position of the reference image and convert to wavelength using the wavelength scale. We convert this wavelength shift to a Doppler shift by using Eqn. 11 with $\lambda = 633$ nm. Finally, we obtain the time observed for each solar spectrum from the header of the .FITS file. We plot the Doppler shift as a function of time and the line of best fit in Fig. 5.

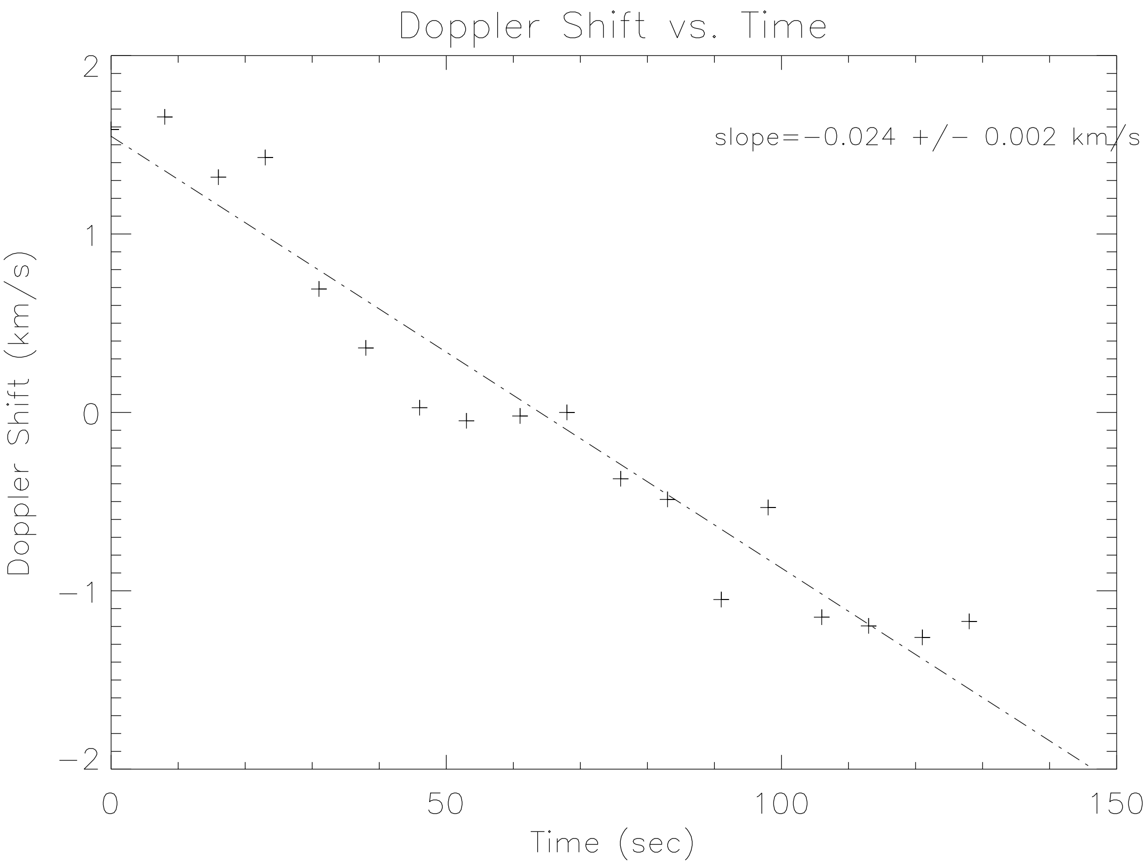

Fig. 5.— The Doppler shift of the Sun as a function of time. The values start off positive and end up negative since one limb of the Sun has a radial velocity component towards the observer while the other limb has a radial velocity component away from the observer. We use the slope of this plot an correct for geometry to find the rotational velocity of the Sun at the equator.

The total transit time of our data run was 128 seconds. Using this and the slope in Fig. 5 we find that rotational speed of the Sun is approximately 1.54 km/sec. This is not the rotational velocity of the Sun at the equator. We must correct since the Sun's axis of rotation is not oriented directly North is our coordinate system.

Define a fixed Cartesian coordinate system where the angular momentum vector of the Sun is in the z-direction with the x-axis pointed towards the observer. The observer can only measure the x component of the rotational speed by obeserving the Doppler shift of light. Now if the spin axis of the Sun is rotated about the y-axis, this will decrease the measured velocity by a factor of $\cos\xi$ where ξ is the angle of rotation. For the case when the spin axis of the Sun is rotated about the x-axis by the angle η, the measured velocity is again decreased by a factor of $\cos\eta$. It is not actually the measured velocity but the rate of change of the measured velocity that decreases. To see this, think of the lines of constant measured velocity when the spin axis is along the z-axis. The lines of constant measured velocity are vertical lines across the disk of the Sun. If the rotation axis is rotated by 90°, the observer would measure no velocity change going across the equator. Thus, the measured change in velocity decreases by $\cos\eta$. For the Sun we have that $\eta = 19.3°$ and $\xi = 2°$. We correct our data by dividing by the cosine of these angles.

We must also correct for the transit time of the Sun. Let ω be the angular velocity of the Earth at the equator and ζ be the angular size of the Sun. If the Sun were directly at the Earth's equator, then the transit time is

$$t_{\text{transit}} = \frac{\zeta}{\omega} \tag{13}$$

Since the Sun is not at the equator of the Earth the transit time increases as latitude increases or decreases. The our transit time of 128 seconds is longer by a factor of $1/\cos(20°)$. Hence, our transit time is 136 seconds. Using this and the tilt angles of the Sun, we find that the rotational velocity of the Sun at the equator is approximately $1.76{\pm}0.08$ km/s. Finally this we can compute the radius of the Sun by

$$R_\odot = \frac{T_{\text{rot}} v_{\text{rot}}}{2\pi} \tag{14}$$

where $T_{\text{rot}} = 26.5$ days. Thus, $R_\odot = 6.4 \times 10^5 \pm 3 \times 10^4$ km. The theoretical value for the rotational velocity of the Sun is 2 km/s and a radius of 7×10^5 km. If we compare our values we find a 9.0% difference for the radius and 13% difference for the rotational velocity.

5. Conclusion

Our results for the rotational velocity and radius of the Sun were slightly out of the acceptable range of error. This mostly likely due to the systematic error that arises from imaging the Sun. We are not certain that the path of our images is exactly along a diameter of the Sun nor are we certain that our first solar image and our last solar image were taken exactly on the limb of the Sun. However, it is remarkble that we were able to measure these properties of the Sun just by investigation the spectrum of light emitted by the Sun.

6. Acknowledgements

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