Photon Counting with a PMT Tube

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ABSTRACT

The detection of light is *the* fundamental tool in astronomy. All of astronomy rests upon the accuracy and precision to which light can be measured and recorded. The electronic detection of light by the photoelectric effect is fundamentally limited by statistical fluctuations. These statistical fluctuations were studied using a photomulitplier tube (PMT) capable of detecting individual photon. The experiment reveals that the detection (counting) of photons is described by Poisson statistics and that increasing the number of measurement increases the accuracy and precision to which one can know the brightness of a source of light.

1. Introduction

Light exhibits its particle-like nature when interacting with matter. A beam of light can be thought of as consisting of many individual particles of light called photons. Each photon has an energy, $E = h\nu$, where h is Planck's constant and ν is the frequency of the light. When light is incident upon a metallic surface, electrons may be ejected in what is known as the photoelectric effect. The ejected electrons have a range of energy with the maximum kinetic energy being

$$K_{max} = E_{\text{photon}} - \phi = h\nu - \phi \tag{1}$$

where ϕ is the work function of the metal (Carroll, 132). Photons can be measured using the photoelectric effect. The ejected electrons constitute a measurable current. However, this method is limited by statistical fluctuations due to the range of energy of the emitted electrons.

The experiment was conducted on September 1, 2005 and September 10, 2005 with partners Sarah Ballard, Megan Reiter, and Vaishali Bhardwa.

1.1. Photomulitplier Tube

The main part of the PMT is the two light emitting diodes (LEDs). One is a constant source of light and acts as a background while the other emits a pulse of light when appropriate and acts as a signal. When a photon from the LED is incident on the cathode an electron is ejected, a current is produced and amplified, and the measurement is recorded digitally. The level of the background LED can be adjusted by a knob labeled CW on the front of the console. Assuming the background level is held constant, there are two other parameters that can be adjusted in the PMT. The sample rate can be set anywhere between 370 and 5000 Hz. The sample rate determines the amount of time the PMT collects data during each sample. If we let Δt be the sample time then

$$\Delta t = \frac{1}{\text{sample rate}}\ \text{sec.} \tag{2}$$

The second parameter that can be adjusted is simply the number of samples. In the experiment, the number of samples is either 100 or 1000.

1.2. The Statistics of Light

For a given sample rate, the number of photons detected per sample varies. The number of photons n, each of energy $E = h\nu$ detected by a detector of area A in the time interval Δt, is important in specifying the intrinsic brightness of a light source

$$I = \frac{P}{A} = \frac{nE}{A} = \frac{nh\nu}{\Delta t \cdot A}. \tag{3}$$

Since h, ν, and A are constant, the brightness of a light source is proportional to $n/\Delta t$. Since n varies with each sample, one can use the mean number of photons per sample as a measure of the brightness of a light source. In this part of the experiment the mean and standard deviation of the number of photons per sample is calculated for several samples to explore the effect of the sample size on these two values. The mean μ and the standard deviation σ are given by

$$\mu = \frac{\sum x_i}{N} \tag{4}$$

and

$$\sigma = \sqrt{\frac{1}{N-1}\sum (x_i - \mu)^2} \tag{5}$$

where N is the number of samples.

1.3. Relationship Between Mean and Standard Deviation

The mean is the average number of photons detected per sample and the standard deviation is a measure of how much each sample value differs from the mean value. If the sample rate is decreased, one would expect the number of photons detected per sample to increase. Intuitively, since more photons are detected, the mean will also increase. What about the standard deviation? In this part of the experiment, we fix the number of samples at 100 and decrease the sample rate from 5000 Hz to 370 Hz to explore the relationship between the mean and standard deviation for photon statistics.

1.4. The Poisson Distribution

The Poisson distribution is a probability distribution that can be used to describe situations involving the counting of random events with a definite average rate.

$$P(x) = \frac{\mu^x}{x!} e^{-\mu} \tag{6}$$

where μ is the average counts per unit time. It seems like Poisson distribution should provide a sound theoretical model of counting photons. This part of the experiment explores the agreement between experimental data and theoretical models to determine if the Poisson distribution or some other probability distribution can be used to model the counting of photons.

1.5. The Standard Deviation of the Mean

One query for data from the PMT provides one set of data where n samples are taken at a fixed rate R. The mean and standard deviation of this set can be calculated as before. One can repeat this 10 times and end up with 10 means and 10 standard deviations. Will all the means be the same? From this, one could calculate the mean of the means (MOM) and the standard deviation of the means (SDOM) of the 10 sets of data. This part of the experiment explores these two quantities because they give in essence the precision to which we can specify the brightness of a light source.

2. Data and Analysis

2.1. The Statistics of Light

To begin the experiment, the PMT is queried for data by specifying the number of samples at 100 samples and the sample rate at 1000 Hz. The data is read into IDL as a 100 element array using the READCOL function. The data is plotted using the IDL PLOT function and it is immediately apparent that the number of photons detected per sample varies.

Five more data sets are obtained of 100 samples each at a sample rate of 1000 Hz. The mean and standard deviation of each of the six sets of data is calculated using Eq. 4 and Eq. 5. The results are listed in the Table. 1 and graphed in Figure. 1.

We repeat the process changing only the sample number per set to 1000. The results are summarized in Table. 2 and graphed in Figure. 2.

Figure. 2 reveals that as the number of samples increases, the shape of the distribution becomes peaked more sharply. At a higher number of samples, the fluctuations in the mean and standard deviation are smaller. This should be expected since Eq. 4 and Eq. 5 reveal that both the mean

Set Number	Mean	Mean Error	Standard Deviation
1	8.04	.3	2.68
2	7.23	.3	3.09
3	8.26	.3	2.89
4	8.05	.3	2.97
5	7.50	.2	2.49
6	7.73	.3	2.89

Table 1: The mean and standard deviation for six sets of data at 100 samples per set at 1000 Hz. There is variation amongst both the mean and standard deviation.

and standard deviation depend on $1/N$ and $[1/(N-1)]^{1/2}$, respectively. If N is large, the deviation of each data point is weighed less heavily.

Set Number	Mean	Mean Error	Standard Deviation
1	8.58	.1	3.04
2	8.60	.1	2.99
3	8.58	.1	2.87
4	8.22	.1	3.00
5	8.18	.1	2.93
6	8.26	.1	2.93

Table 2: The mean and standard deviation for six sets of data at 1000 samples per set at 1000 Hz.

2.2. Relationship Between the Mean and Standard Deviation

The number of samples is fixed at 100 and the sample rate is varied from 5000 Hz to 370 Hz. The mean and standard deviation for each set of data is calculated using Eq. 4 and Eq. 5. The mean of each set and the square of the standard deviation of each set or the variance is plotted using the IDL PLOT function. The mean vs. mean curve is plotted on the same axes using the IDL OPLOT function and is shown in Figure. 3.

From the graph, it may be deduced that

$$\mu = \sigma^2 \tag{7}$$

since there appears to be a linear relationship between the mean and variance. In Poisson statistics, the standard deviation is in fact equal to the square root of the mean. This partial experimental confirmation that photon counting follow Poisson statistics.

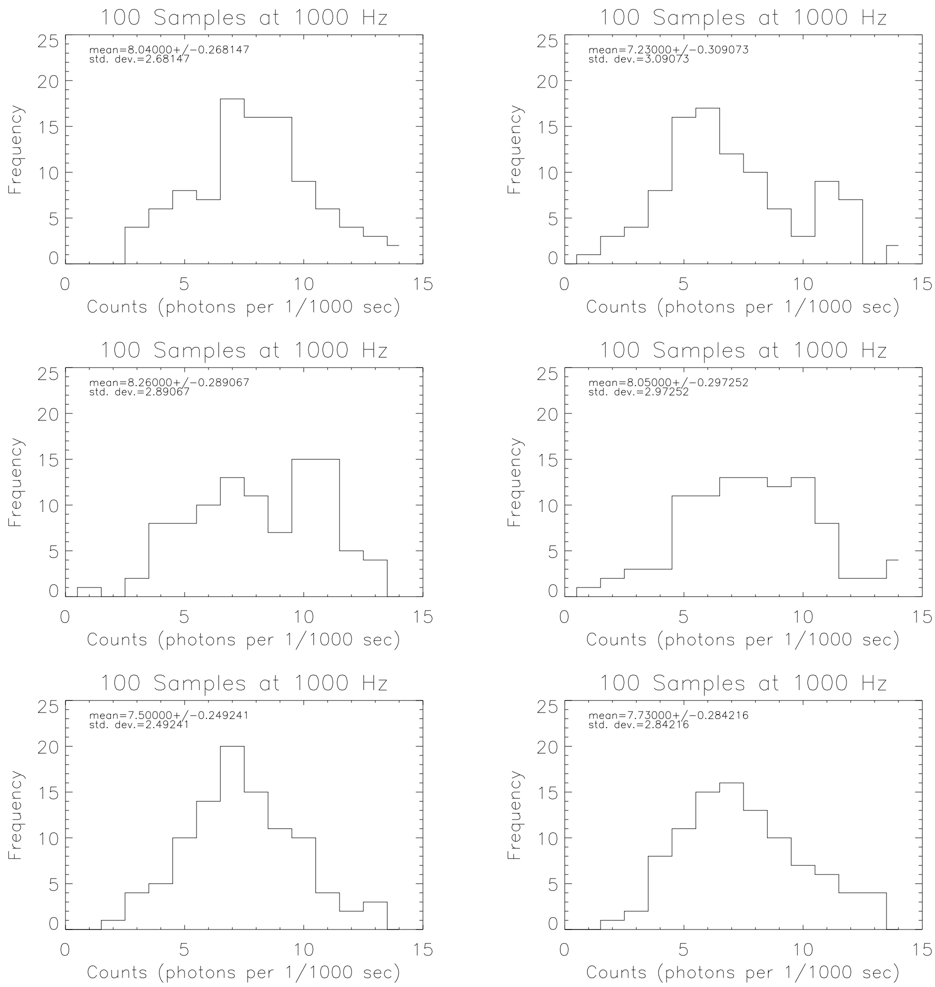

Fig. 1.— Six histograms of six sets of 100 samples at 1000 Hz. It is clear that the mean and standard deviation for each set is not equal

2.3. The Poisson Distribution

The Poisson distribution is graphed against a set of 1000 samples at 5000 Hz in Figure. 4 and Gaussian distribution,

$$G(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \tag{8}$$

where σ is the standard deviation, is graphed against a set of 1000 samples at 370 Hz in Figure 5. Both the Poisson distribution and the Gaussian distribution must be scaled to fit the data. Since the two are probability distributions, their range is between 0 and 1. Thus, to scale the data we multiply by the number of elements, which is in this case is 1000.

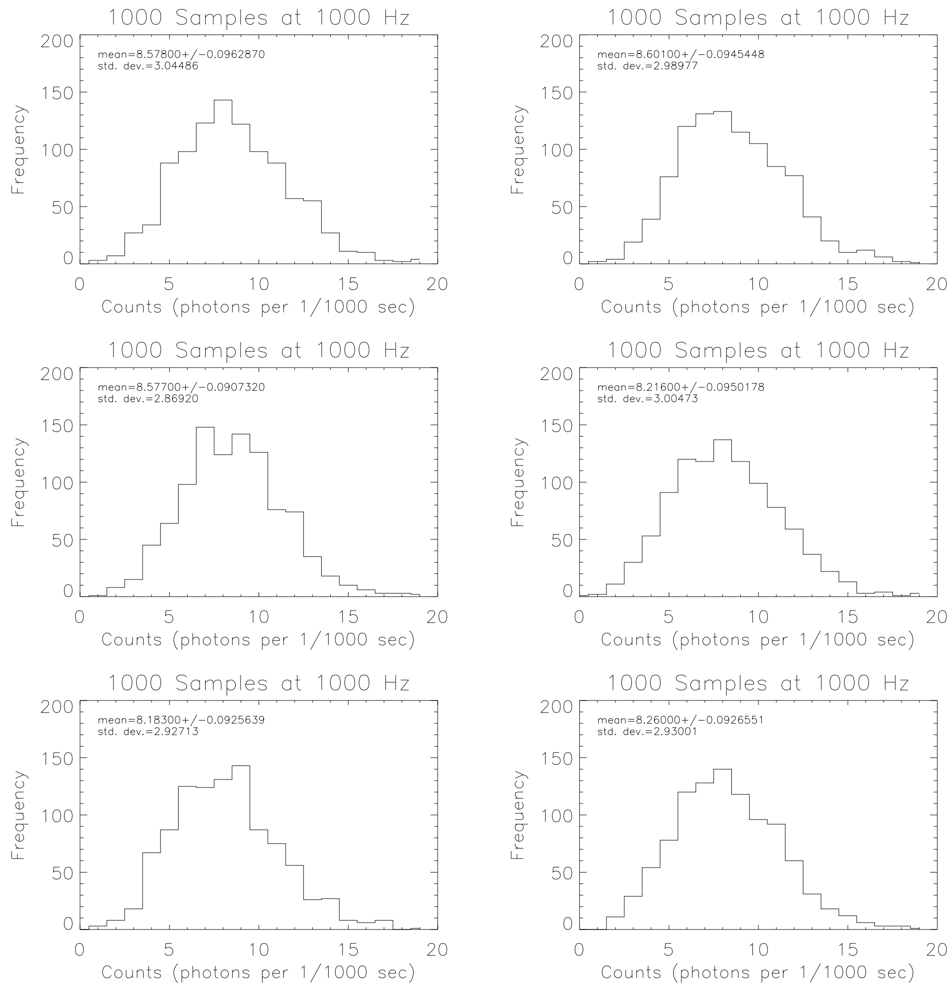

Fig. 2.— Six histograms of six sets of 1000 samples at 1000 Hz. There is fluctuation in both the mean and standard deviation but the distribution have become more sharply peaked about the mean.

The Poisson distribution is used to fit one set of data and the Gaussian is used to fit another. The only parameter changed between the two data sets is the sample rate. At 5000 Hz, around 10 photons are counted per sample whereas at 370 Hz, around 250 photons are counted per sample. The main difference between the two sets is the mean, which is higher for 370 Hz. This reveals that the Gaussian distribution is a good approximation to the Poisson distribution for large μ.

Since the detection of a photon is random, the distribution of photons per sample should be symmetric about the mean. Hence, the Gaussian distribution should provide a good theoretical model. However, if the mean of the photons per sample is close to zero, then the left end tail of the distribution must decrease more rapidly than the right end tail because it is insensible to talk

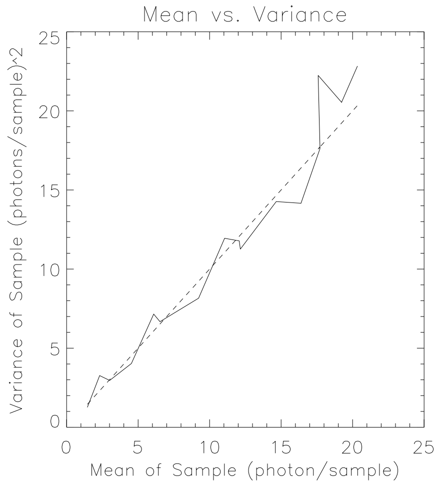

Fig. 3.— A plot of the mean vs. variance and mean vs. mean when the sample number is held fixed and the sample is varied from 5000 Hz to 370 Hz

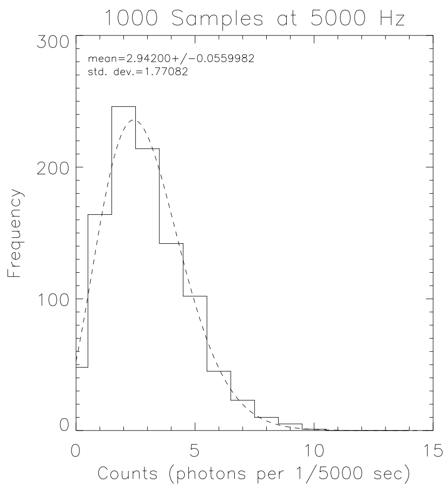

Fig. 4.— A histogram of a set of 1000 samples at 5000 Hz and the corresponding Poisson distribution. Clearly, the Poisson Distribution is a good approximation to the experimental data for small μ.

about counting negative photons. Hence, Poisson statistics can be used to model photon counting in general, but becomes the Gaussian distribution in the limit of large μ.

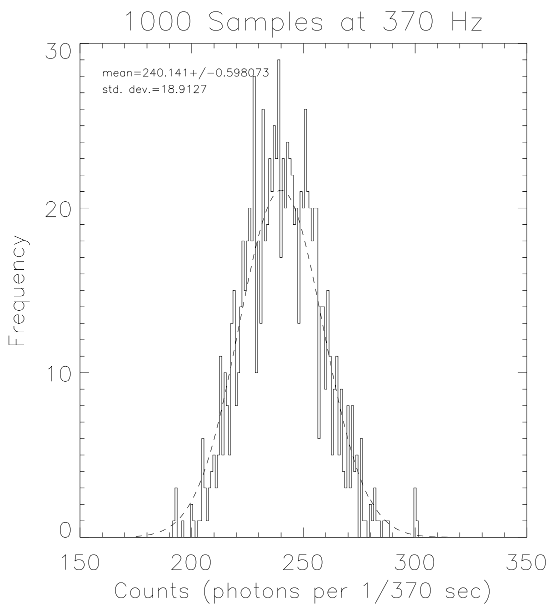

Fig. 5.— A histogram of a set of 1000 samples at 370 Hz and the corresponding Gaussian distribution. The Gaussian distribution fits the experimental data well when the mean is large.

2.4. The Standard Deviation of the Mean

The sample rate is fixed at 1000 Hz and 10 sets of data are taken at different sample sizes. Then mean and standard deviation is calculated for each of the 10 data sets for a given sample rate using Eq. 4 and Eq. 5. The mean of the means and the standard deviation of the means is calculated using the 10 means for each sample size. The data is summarized in Table. 3 and a MOM vs. sample number and SDOM vs. sample number plots are shown in Figure. X and Figure. X.

From the graphs, we see that there is some strong variation in the MOM when the sample number is small. This is no surprise considering when you have less data, each deviation is weighted more heavily. But as the number of samples increase, the MOM curve stays relatively constant. The SDOM decrease noticeable with an increase in sample size. This is exactly the opposite of what is occurring for the MOM when sample size is small. As the sample size gets larger, there is still variation amongst the data. However, each deviation is weighted less heavily because of the factor of $1/N$ and $[1/(N-1)]^{1/2}$, respectively, in Eq. 4 and Eq. 5. Hence, the measurement of the MOM is more precise for larger sample number.

With knowledge of Poisson statistics, it is possible to calculate the SDOM from the mean and

Sample Number	MOM	MOM Error	SDOM
2	15.90	0.8	2.60
4	16.40	0.6	1.80
8	15.94	0.6	1.86
16	15.31	0.3	0.87
32	15.90	0.2	0.55
64	15.90	0.2	0.58
128	15.95	0.1	0.36
256	16.09	0.1	0.41
512	16.14	0.1	0.21
1024	16.13	0.03	0.10
2048	16.48	0.03	0.09

Table 3: The MOM and the SDOM for 10 sets of data at 1000 Hz of various sample size. The MOM stays fairly constant while the SDOM decreases with increasing sample number.

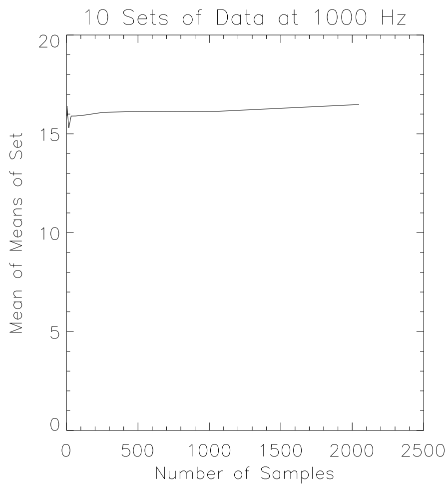

Fig. 6.— A graph of the MOM vs. sample number. There is some strong variation in the MOM for small sample size but the curve smooths out for large sample size.

sample size.

$$\sigma_\mu = \frac{\sigma}{\sqrt{N}} = \sqrt{\frac{\mu}{N}} \tag{9}$$

since in Poisson statistics $\mu = \sigma^2$ (Taylor, 102). Thus, to improve the accuracy of a measurement of the mean by a factor of two, one must increase the sample size by 4.

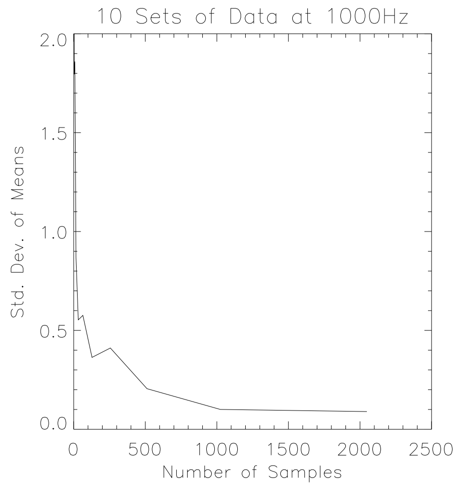

Fig. 7.— A graph of the SDOM vs. sample number. It decreases with an increase in sample size. This shows that the more samples you take, the more precise the data will be around the mean.

3. Conclusions

The detection of light is a process with statistical limitations in regards to accuracy and precision of measurement. For a given time interval, the most accurate way to quantify the brightness of a light source is by the mean number of photons detected from the source. If another measurement is made for the same time interval on the same source, the experiment is likely to yield a different result. However, as the number of measurements increases, the standard deviation decrease and there is less significant fluctuations from the mean. In taking more measurements, each individual measurement has less weight in the overall average. Thus, to be more precise about the brightness of a source of light, more measurements must be taken.

There is enough experimental evidence to conclude that Poisson statistics is the appropriate theoretical model for the counting of light. The experiment found that when counting photons, for a given sample size and sample rate, the mean equals the square of the standard deviation. The error in the mean is the standard deviation of the means. From Poisson statistics, the error in the mean is given by Eq. 9. If sample size and mean are known, the error is easily calculated. Furthermore, when sample size is increases, standard deviation also decreases. From Eq. 9, we see that as sample size increases the error in the mean also decreases. Hence, to obtain the most accurate measurement of the brightness of a source of light, take the most number of measurements possible.