A Gentle Introduction to Numerical Simulations of Galaxy Mergers

Christopher Q. Trinh
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ABSTRACT

This paper should provide the reader with a clear starting point for doing numerical simulations of galaxy mergers. The equations of motion for the different components of a galaxy are discussed in general. The specific methods used to integrate these equations forward in time are presented, along with some advantage and disadvantages of each method.

1. Introduction

It is well-known that the Andromeda galaxy, M 31, is the nearest neighbor to the Milky Way. Despite the expansion of the universe, gravitational forces are pulling the two galaxies closer and closer. The stellar population and interstellar gas in the galaxies begin to interact strongly as they approach each other. Depending on the geometry of the encounter, the stars and gas may form bridges that connect the two galaxies or tails that extend far away from the individual galaxy or even other exotic fine structures. If the two galaxies pass close enough, their orbits will decay as orbital angular momentum is transferred to internal rotation and they will eventually merge into a single tightly bound system.

Observationally, there are many dramatic examples of recent or ongoing interactions between galaxies. The most famous example of an interacting system is M 51 and its companion NGC 5195. The two galaxies show clear signs of tidal damage. M 51 displays a very magnificent spiral structure and is linked to NGC 5195 by a distinct bridge. Another example of an interacting system is NGC 4676, also known as the Mice. Each spiral galaxy possess a tail. The two tails are very different. One of the tails is long and straight whereas the other is curved like a ribbon. Also, the straight tail is narrow and bright whereas the curved tail is faint and diffuse. Another great example is the Antennae, NGC 4038 and NGC 4039. The two galaxies have collided and the system contains two narrow tails extending far from the colliding nuclei. Images of these systems can be found on the NASA Extragalactic Database at http://nedwww.ipac.caltech.edu/.

These examples of interactions are a few of those observed during the present epoch. However, the interaction represents only a short phase in the overall lifetime of a galaxy. Toomre (1977) used a very general argument to show that significant merging must have occurred during the formation of most galaxies. These two ideas indicate that there exists galaxies that have gone through an interaction and settled into a structure that exhibits only subtle scars from the interaction in the far past. Thus, the interaction of galaxies may be thought of as a fundamental evolutionary process and a better understanding of such a process will provide insight into the formation of structures seen today. In addition, the study of interacting galaxies is motivated by the observational and theoretical links between interactions and bursts of star formation and the generation of unusual forms of energy in galaxies.

2. Galactic Structure and Dynamics

2.1. Hubble Sequence

The Hubble sequence is a classification scheme developed by Edwin Hubble, which classifies a galaxy based on its shape or morphology. Most of the galaxies observed today are either elliptical galaxies (E) or spiral galaxies (S). Elliptical galaxies tend to be dense and have little to no rotation. Each elliptical galaxy has an associated eccentricity, which is the ratio of the semi-major to the semi-minor axis. Elliptical galaxies with an eccentricity of zero are spheres and those with an eccentricity of 0.7 are very elongated. Spiral galaxies have diffuse disk with varying degree of spiral structure surrounding a central bulge. These tend to have rapid rotation about the galactic center. The central bulge may or may not have a central bar. Those with a bar are denoted by (SB). Initially, the Hubble sequence was thought to be an evolutionary sequence for galaxies. In this model, galaxies began as elliptical galaxies with small eccentricity. As these elliptical galaxies evolved their eccentricity increased toward 1. Eventually, elliptical galaxy would flatten-out so much that it would essentially become a barred or unbarred spiral galaxies. These spiral galaxies had tightly wound spiral arms that fanned out at time progress. It is now known that the Hubble sequence is *not* an evolutionary sequence for galaxies. Yet, it is intriguing that most galaxies can roughly be classified as one of two morphologies.

2.2. Equations of Motion

We have discussed what galaxies look like, but what are they made of? Galaxies are gravitationally bound systems of stars and gas surrounded by a dark matter halo. The number of stars in a given galaxy is typically between 10^7 and 10^{12}. The gas in galaxies is known as the interstellar medium and is approximately 99% gas and 1% dust by composition. The total mass of the various phases of the ISM is approximately 10% of the total mass of the luminous component in a given galaxy. The individual elements that comprises each component evolves in time. In order to follow the time evolution of galaxies, we must known the appropriate initial conditions, i.e., the initial position and velocity of each element, and also have an equation of motion for each component.

In a given galaxy, there is a gravitational potential $\Phi(\boldsymbol{x}, t)$. The gravitational potential determines the acceleration of a mass element at every point at every instant in time. It is the quantity that is causing changes in motion. The gravitational potential is given by the Poisson equation

$$\nabla^2 \Phi(\boldsymbol{x}, t) = 4\pi G \rho(\boldsymbol{x}, t) \tag{1}$$

where G is the gravitational constant and $\rho(\boldsymbol{x}, t)$ is the total mass density from all components.

To completely describe the state of the interstellar gas, we must know the mass density $\rho_g(\boldsymbol{x}, t)$, the pressure $\mathcal{P}(\boldsymbol{x}, t)$, and the velocity $\boldsymbol{v}(\boldsymbol{x}, t)$. The equations of motion are simply the ordinary equations for a compressible fluid (see e.g., Landau & Lifschitz 1965). The first equation is a statement of the conservation of mass and is known as the continuity equation,

$$\frac{\partial \rho_f}{\partial t} + \nabla \cdot (\rho_g \boldsymbol{v}) = 0. \tag{2}$$

The second equation describes the time variation of the velocity of the fluid and is known as the Euler equation,

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{1}{\rho_g}\nabla \mathcal{P} - \nabla \Phi. \tag{3}$$

The third equation describes the dissipation of energy in a collisional fluid,

$$\rho_g \frac{\partial u}{\partial t} + \rho_g\,(\boldsymbol{v} \cdot \nabla)u + \mathcal{P}\,(\nabla \cdot \boldsymbol{v}) = -\mathcal{L} \tag{4}$$

where u is the thermal energy per unit mass, and $\mathcal{L}$ is the energy loss function that comprises all non-adiabatic sources and sinks of energy (see e.g., Barnes and Hernquist 1996). These three equations must be closed by an equation of state that usually relates the pressure and density. The exact equation of state depends on the physical process in the fluid. Two examples are the ideal gas and the isothermal equations of state,

$$\mathcal{P} = (\gamma - 1)\rho_g\, u \tag{5}$$

$$\mathcal{P} = \rho_g\, v_s^2 \tag{6}$$

where γ is the ratio of specific heat at constant pressure and constant temperature, and v_s is the isothermal sound speed.

The baryonic stellar population is most conveniently described in phase-space by a distribution function $f(\boldsymbol{x}, \boldsymbol{v}, t)$. Each star is associated with a six-dimensional phase-space vector plus a time component and the number of stars with position between $\boldsymbol{x}$ and $\boldsymbol{x} + d\boldsymbol{x}$ and velocity between $\boldsymbol{v}$ and $\boldsymbol{v} + d\boldsymbol{v}$ is $f(\boldsymbol{x}, \boldsymbol{v}, t)\, d^3\boldsymbol{x}\, d^3\boldsymbol{v}$. Since stars in given galaxy are very unlikely to collide with each other because the relaxation time is large compared to Hubble time (see e.g., Binney and Tremaine 1987), the stellar population can be treated as a *collision-less* system and the phase-space density of stars is conserved, i.e., the stars drift smoothly through space. The equation of motion of the stellar component is known as the collision-less Boltzmann equation,

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla f - \nabla \Phi \cdot \frac{\partial f}{\partial \boldsymbol{v}} = 0. \tag{7}$$

If instead the stellar component were a *collisional* system, there would be additional terms on the right-hand side of Eq. (7). It is believed that the dark matter component also obeys the collision-less Boltzmann equation (see e.g., Dodelson 2003).

The coupled equations of motion for the components of a galaxy given in Eqs. (1) through (7) can only be solved analytically if a high level of symmetry or if simplifying assumptions are made. In order to study interacting galaxies in general numerical simulations must be used.

3. Numerical Simulations

3.1. Gravity Solvers

3.1.1. Finite-Difference Techniques

Numerical simulations can be used to integrate the equations of motion for the components of a galaxy forward in time. The most direct way to do this is known as the finite difference technique. In this algorithm, the initial conditions are specified at some time t_0. With these initial conditions, the equations of motion are used to find the value of each parameter at a new time $t_0 + \Delta t$. This process is repeated to find the value of the parameters at time t_{i+1} given that the values at time t_i is known or has been computed.

In general, the use of a finite-difference method is extremely prohibitive. The problem lies in the extremely large number of stars, gas particles, and dark matter elements that reside in a given galaxy. Each equation of motion depends on the gravitational potential at time t, since it is the gravitational potential that is causing changes in motion. If the galaxy is composed if N elements the most straightforward method of computing the gravitational potential at mass element k at time t is the direct summation of the gravitational potential from every other mass element. This must be done at every time step for every particle. If the number of elements is large, as it is in galaxies, this method is too slow because the the CPU cost per step scales at $\sim \mathcal{O}(N^2)$.

3.1.2. Particle-Mesh Algorithm

There exists other potential solvers, each with their own advantages and disadvantages. One example is known as the particle-mesh (PM) method. The PM method divides space up into a finite Cartesian grid with a fixed number of points. Each mass element is assigned to one point on the grid. The number of mass elements at each point on the grid is then proportional to the mass density at that grid point. To find the gravitational field at each grid point first notice that in Fourier space

$$\tilde{\Phi} = -4\pi G \frac{\tilde{\rho}}{k^2}. \tag{8}$$

where $\boldsymbol{k}$ is the comoving wavenumber. Multiplying by $\boldsymbol{k}$ and computing the inverse Fourier transform gives the gravitational field at each grid point. The PM method is more efficient than direct summation as the CPU cost per step scales as $\sim \mathcal{O}(N) + \mathcal{O}(N_g \log N_g)$, where N_g is the number of grid points. Clearly, the resolution of the PM method is limited by the number of grid points. As it turns out, the inverse square law is not well reproduced when particles are very close on the grid (see e.g., Bertschinger 1998). Hence, the PM method may not be a suitable gravity solver for the numerical simulation of interacting galaxies since, as we will see, the interaction tends to drive the interstellar gas into the center of the galaxy producing a high density nuclear gas.

3.1.3. Barnes-Hut Tree Algorithm

There is another potential solver introduced by Barnes & Hut (1986) that has a CPU cost per step that scales as $\sim \mathcal{O}(N \log N)$. At each step of the algorithm before the gravitational potential is computed at the position of a particle, space is divided up recursively into a hierarchy of cells and subcells. This is repeated until each cell either contains one or zero particles. The mass, center of mass, and a low-order multipole moment of each cell is computed and stored. To compute the gravitational potential at the position of a given particle, the hierarchy of cells is examined from the top down. At each step, the size of the cell s is compared with its distance d to see if the two satisfy the condition $s/d < \theta$, where θ is a free parameter. If the condition is satisfied, then all particles in that cell are treated as a single particle at the center of mass and the contribution of the aggregate to the potential is the low-order multipole moment. This is how the Barnes-Hut Tree algorithm is able to save some computation time. The elements near the position where the gravitational potential is being computed are directly summed while particles that are far way are lumped into a single particle at their center of mass. Since the method is gridless it does not impose any artificial restrictions on the global geometry or any limitations on the dynamic range in spatial resolution as in the PM method (see e.g., Hernquist & Katz 1988). Thus, the Barnes-Hut Tree algorithm is more suited to the numerical simulation of interacting galaxies since it can adapt the spatial resolution as the gas density in the interacting galaxies increases.

At this point, we have covered all the simulation algorithms necessary to integrate the collisionless Boltzmann equation forward in time. The Barnes-Hut Tree algorithm can be used to compute efficiently the gravitational force on the i^{th} star or dark matter particle at each time step. Then single Boltzmann equation is equivalent to two first-order differential equations

$$\frac{d\boldsymbol{x}_i}{dt} = \boldsymbol{v}_i \tag{9}$$

$$\frac{d\boldsymbol{v}_i}{dt} = -\nabla\Phi \tag{10}$$

which should be recognized as Newton's Second Law. It makes intuitive sense that the equations of motion are quite simple here since the stars and dark matter are collision-less systems.

3.2. Gas Dynamics

Although the gravitational force on each particle at each time step can be computed efficiently the interstellar gas is a collisional system and the equations of motion are much more complicated than the collision-less Boltzmann equation governing the stellar component and the dark matter halo in galaxies. Thus, numerical integration of these equations also require great care and ingenuity.

The most widely used computational method for simulating fluid flows is known as smoothed particle hydrodynamics (SPH), which is gridless and Lagrangian in nature. The starting point for SPH is representing the continuous fluid as a collection of N fluid elements. A real fluid would be composed of an infinite number of these elements with each being infinitesimal extent. The mass density of the finite fluid elements is proportional to the real fluid mass density ρ_g (see e.g., Gingold & Monaghan 1977). The average value of the real fluid mass density can be approximated from smoothed average of the local density of fluid elements over a finite volume.

The two fluid equations that need to be integrated forward in time are Eqs. (2) and (3). Since SPH is Lagrangian, the continuity equation is automatically satisfied and only the Euler equation needs to be integrated. In order to integrate the Euler equation forward in time, we need to be able to compute the mass density of the fluid, the gradient of the pressure of the fluid, and also the gradient of the velocity of the fluid. How are these quantities computed in SPH? As stated before, the mass density can be approximated from the average value of the local mass density of the fluid elements over some finite volume. In fact, any physical parameter of the real fluid can be estimated in this way. The size of the averaging volume is determined by the smoothing length h. To compute the average value of the physical field, SPH requires a normalized smoothing kernel function W to correct for statistical fluctuations in the fluid element number. Thus, the mean value of the physical field within a given interval is

$$\langle f(\boldsymbol{x})\rangle = \int W(\boldsymbol{x}-\boldsymbol{x}';h)f(\boldsymbol{x}')\,d\boldsymbol{x}'. \tag{11}$$

Commonly used smoothing kernel functions are the Gaussian function and the cubic spline. We only know $f(\boldsymbol{x})$ at a finite number of discrete points with number density $n(\boldsymbol{x}) = \sum_{i=1}^{N} \delta(\boldsymbol{x}-\boldsymbol{x}')$. Thus, the integral in Eq. (11) becomes

$$\langle f(\boldsymbol{x})\rangle = \sum_{i=1}^{N} \frac{f(\boldsymbol{x}_i)}{\langle n(\boldsymbol{x}_i)\rangle}\, W(\boldsymbol{x}-\boldsymbol{x}_i;h). \tag{12}$$

Furthermore, the gradient of the physical field $f(\boldsymbol{x})$ can also be found by averaging over some finite volume and is given by

$$\langle \nabla f(\boldsymbol{x})\rangle = \sum_{i=1}^{N} \frac{f(\boldsymbol{x}_i)}{\langle n(\boldsymbol{x}_i)\rangle}\, \nabla W(\boldsymbol{x}-\boldsymbol{x}_i;h). \tag{13}$$

This is the key to SPH. To summarize, SPH first partitions the fluid into a finite number of discrete fluid elements with a mass density proportional to the mass density of the real fluid. Then any physical field can be estimated by averaging over a finite volume determined by the smoothing length with a normalized smoothing kernel according to Eqs. (12) and (13).

4. Interacting Galaxies

Now that some common numerical algorithms that been discussed, we turn to the discussion of how these techniques have been implemented by different groups to study interacting galaxies and the results these simulations produced.

4.1. Tidal Interactions

Believe or not, the idea that gravitational forces alone could produce the range of fine structures that are observed in interacting galaxies was not always accepted. In fact, it was not until Toomre & Toomre (1972) performed a very simplistic N-body simulation of interacting galaxies was it shown that tidal interactions can produce the bridge the connects M 51 and NGC 5195 or the two tails of NGC 4676. Their numerical simulation consisted of two test particles of some mass. One of the point masses was surrounded by several rings of massless test particles while the other was left bare. With these simple models for galaxies, they ran four simulations where the orbital geometry was restricted to either a prograde and retrograde pass in the plane of rotation of the disk of test particles. Their simplistic simulations showed the tidal disturbance was quite mild for equal mass galaxies in a retrograde passage. However, a flat parabolic, prograde passage of an equal mass galaxy produced a transient bridge and a long, diffuse, curving tail. Under the same conditions except reducing the mass of the perturbing galaxy to 1/4 produces a well-defined bridge and counter-arm. Lastly, the flat, parabolic, prograde passage a perturbing galaxy 4 times as massive again produces a long, diffuse, curving tail. Again, to emphasize, this paper elevated the importance of tidal interactions. However, the models for galaxies here were not self-consistent, i.e., they did not include self-gravity. It might have been acceptable for a preliminary study. However, a true understand of the dynamics of interacting galaxies must include self-gravity.

Although there were probably simulations done earlier that included self-gravity, the paper by Barnes (1991) mostly uses code similar to the those that have been discussed. This paper presented the results of numerical simulations of mergers between equal mass disk galaxies with self-gravity but no interstellar gas. It was done to see if a detailed analysis of merger remnant properties could put a constraint on the number of elliptical galaxies produced by mergers of disk galaxies. While this was not achieved it was found that the most extended components of two galaxies interact tidally. The nature of this interaction of to transform orbital angular momentum into internal rotation. The tightly bound components are not directly affected by tidal interactions. However, the central regions are perturbed when their forward motion is impeded by their halos, which *are* being affected by tidal effects. In the next paper we discuss, the loss of orbital angular momentum will induce bursts of star formation in interacting galaxies.

4.2. Starburst Galaxies

Larson & Tinsley (1977) showed that galaxies in the Arp (1966) atlas had the tendency to be bluer when compared to isolated counterparts. There is support that this phenomenon is due to intense star formation. Numerical simulations were used by Mihos & Hernquist (1995) to study the "development of gaseous inflows and triggering of starburst activity in the merger of disk galaxies of comparable mass." This was done using a code called TREESPH, which is the combination of the Barnes-Hut Tree algorithm and SPH. The study included a range of orbits and it was found that in each case that the interaction forced a large percentage of the gas in each galaxy to fall into the center due to the loses angular momentum gravitationally by the gas. The time at which the inflow occurs during the interaction and the intensity seems to be sensitive to the structure of the interacting galaxies. Galaxies with dense central bulges have delayed but strong inflow during the later stages of merging. The dense central bulge stabilizes the galaxy against bar mode and inflow until they finally merge. On the other hand, galaxies lacking a dense central bulge experience a central concentration of gas much earlier and much weaker. The results also support the notion that a merger between disks galaxies produces an elliptical galaxy. While this simulation seemed to be quite comprehensive in that the interacting galaxies included a stellar component, interstellar gas, and a dark halo with self-gravity, there is still more physics that can be incorporated to more realistically model the interaction of galaxies. An example is a multi-phased interstellar medium. Nevertheless, it is quite amazing that these computations can be done at all.

5. Conclusion

This paper was meant as a primer to numerical simulations of galaxy mergers. The equations of motion of each component in a galaxy have been discussed, as well as some numerical techniques to integrate these equations forward in time in order to follow the time evolution of the separate components in a galaxy. Some specific papers that involve simulations of interacting galaxies have been discussed as well as a few of the interesting results produced by these simulations. We have not made an effort to present more details about the implementation of algorithms because it is beyond the scope of this gentle introduction. The interested reader is encouraged to browse the references. We hope that the reader has gained an appreciation of the power of numerical simulations and the importance of galaxy mergers in the evolution of the universe.

6. References

Barnes, J.E., Hut, P. 1986. Nature. 324:446

Barnes, J.E., Hernquist, L. 1991. Annu. Rev. Astron. Astrophys. 1992. 30:705

Barnes, J.E. 1992. Ap. J. 393:484

Barnes, J.E., Hernquist, L. 1996. Ap. J. 471:15

Bertschinger, E. 1998. Annu. Rev. Astron. Astrophys. 36:599

Binney, J., Tremaine, S. 1987. *Galactic Dynamics*. Princeton: Princeton Univ. Press

Dodelson, S. 2003. *Modern Cosmology*. San Diego: Academic Press

Gingold, R.A., Monaghan, J.J 1977. Mon. Not. R. astr. Soc. 181:375

Hernquist, L., Katz, N. 1988. Ap. J. 70:419

Landau, L.D., Lifshitz, E.M. 1959. *Fluid Mechanics*. Oxford: Pergamon

Mihos, J.C., Hernquist, L. 1995. Ap. J. 464:641

Toomre, A., Toomre, J. 1972. Ap. J. 178:623

Toomre, A., 1977. Ann. Rev. Astron. Astrophys. 15:437