2D BRUTE-FORCE FITTING FOR LAB 3 BASELINE DETERMINATION
February 28, 2017

This is a recap of the recommended 'brute-force' fitting procedure to find the baselines B_{ew} and B_{ns}, which is equivalent to finding Q_{ew} and Q_{ns} (defined in the first few lines of section 8.4 in lab3 writeup).

1. Follow the first few lines of section 8.4.3 to find the best value for Q_{ew} assuming that $Q_{ns} = 0$. That is, generate a series of 'guessed values' for Q_{ew}. For each one, least-squares fit for the unknown coefficients (A, B) in eqn 12 and derive the sum-of-squared residuals S^2. Plot S^2 vs Q_{ew}; the least-squares fit for Q_{ew} is that value for which S^2 is minimized.

Issues: Cover a large enough range of Q_{ew} in your guesses so that you find the true minimum, not just a local minimum. Also, make the guessed values close together enough so that you define the minimum well.

2. Repeat the above for a 2-d grid of guessed values for Q_{ew} and Q_{ns} and find the global minimum for S^2 in this 2d space. Call this minimum value S^2_{min}.

The values for Q_{ew} and Q_{ns} at this global minimum are the best-fit values. Call these $Q_{ew}*$ and $Q_{ns}*$. Define $\Delta Q_{ew} = Q_{ew} - Q_{ew}*$ and $\Delta Q_{ns} = Q_{ns} - Q_{ns}*$.

3. Deriving the uncertainties in the best-fit values:

3.1. Convert your 2-d grid of S^2 into a 2-d grid of ΔS^2, where $\Delta S^2 = S^2 - S^2_{min}$. Then the Taylor expansion about the best-fit values begins with the second-order terms because the first-order ones are zero at the minimum. So with the second derivatives evaluated at S^2_{min}, we have

$$\Delta S^2 = \frac{1}{2}\frac{\partial^2 S^2}{\partial Q_{ew}^2}\Delta Q_{ew}^2 + \frac{\partial^2 S^2}{\partial Q_{ew}\partial Q_{ns}}\Delta Q_{ew}\Delta Q_{ns} + \frac{1}{2}\frac{\partial^2 S^2}{\partial Q_{ns}^2}\Delta Q_{ns}^2 \tag{1}$$

It's much better to write this in matrix notation:

$$\Delta S^2 = \mathbf{\Delta Q^T} \cdot [\alpha] \cdot \mathbf{\Delta Q} \quad , \tag{2}$$

where $\mathbf{\Delta Q}$ is the column vector of guessed values minus the best-fit values, i.e.

$$\mathbf{\Delta Q} = \begin{bmatrix} \Delta Q_{ew} \\ \Delta Q_{ns} \end{bmatrix} \tag{3}$$

and $[\alpha]$ is the curvature matrix (equation 2.4a in 'least-squares-lite'), which is 2x2 and symmetric so it has 3 independent parameters—2 diagonal and 1 off-diagonal element:

$$[\alpha] = \begin{bmatrix} \frac{1}{2}\frac{\partial^2 S^2}{\partial Q_{ew}^2} & \frac{\partial^2 S^2}{\partial Q_{ew}\partial Q_{ns}} \\ \frac{\partial^2 S^2}{\partial Q_{ew}\partial Q_{ns}} & \frac{1}{2}\frac{\partial^2 S^2}{\partial Q_{ns}^2} \end{bmatrix} \tag{4}$$

3.2. Extract approximate values for the elements of $[\alpha]$ by hand. Use your native intelligence to do this in an easy way. Specifically, use your 2d grid of ΔS^2 versus Q_{ew} and Q_{ns} to numerically find the three second derivative matrix elements.

3.3. Having found $[\alpha]$, find the covariance matrix by taking its inverse.

3.4. Derive the uncertainties in Q_{ew} and Q_{ns} using this covariance matrix in equation 3.7 in 'lsfit-lite'.