Now the D transformation is a rotation about z, and hence has a matrix of the form (cf. Eq. (4–17)):

$$\mathsf{D} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{4–43}$$

The C transformation corresponds to a rotation about ξ, with the matrix:

$$\mathsf{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}, \tag{4–44}$$

and finally B is a rotation about ζ' and therefore has the same form as D:

$$\mathsf{B} = \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{4–45}$$

The product matrix $\mathsf{A} = \mathsf{BCD}$ then follows as

$$\mathsf{A} = \begin{pmatrix} \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi & \cos\psi\sin\phi + \cos\theta\cos\phi\sin\psi & \sin\psi\sin\theta \\ -\sin\psi\cos\phi - \cos\theta\sin\phi\cos\psi & -\sin\psi\sin\phi + \cos\theta\cos\phi\cos\psi & \cos\psi\sin\theta \\ \sin\theta\sin\phi & -\sin\theta\cos\phi & \cos\theta \end{pmatrix}. \tag{4–46}$$

The inverse transformation from body coordinates to space axes

$$\mathbf{x} = \mathsf{A}^{-1}\mathbf{x}'$$

is then given immediately by the transposed matrix $\tilde{\mathsf{A}}$:

$$\mathsf{A}^{-1} = \tilde{\mathsf{A}} = \begin{pmatrix} \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi & -\sin\psi\cos\phi - \cos\theta\sin\phi\cos\psi & \sin\theta\sin\phi \\ \cos\psi\sin\phi + \cos\theta\cos\phi\sin\psi & -\sin\psi\sin\phi + \cos\theta\cos\phi\cos\psi & -\sin\theta\cos\phi \\ \sin\theta\sin\psi & \sin\theta\cos\psi & \cos\theta \end{pmatrix}. \tag{4–47}$$

Verification of the multiplications and of the orthogonality of the A matrix will be left as an exercise.

4–5 The Cayley-Klein parameters. Various other groups of variables have been used to describe the orientation of a rigid body, usually for convenience in some specialized calculation. One such is of sufficient interest to deserve further mention — the so-called *Cayley-Klein parameters*. As there are four quantities involved, these parameters are not all independent, and therefore are not suitable as generalized coordinates. Originally, they were introduced into classical mechanics by Felix Klein