$\xi'\eta'\zeta'$ axes. The ξ' axis is at the intersection of the xy and $\xi'\eta'$ planes and is known as the line of nodes. Finally the $\xi'\eta'\zeta'$ axes are rotated counterclockwise by an angle ψ about the ζ' axis to produce the desired $x'y'z'$ system of axes. Fig. 4–6 illustrates the various stages of the sequence. The Eulerian angles θ, ϕ, and ψ thus completely specify the orientation of the $x'y'z'$ system relative to the xyz and can therefore act as the three needed generalized coordinates.*

The elements of the complete transformation A can be obtained by writing the matrix as the triple product of the separate rotations, each of which has a relatively simple matrix form. Thus, the initial rotation about z can be described by a matrix D:

$$\boldsymbol{\xi} = \mathsf{D}\mathbf{x},$$

where $\boldsymbol{\xi}$ and $\mathbf{x}$ stand for column matrices. Similarly the transformation from $\xi\eta\zeta$ to $\xi'\eta'\zeta'$ can be described by a matrix C:

$$\boldsymbol{\xi}' = \mathsf{C}\boldsymbol{\xi},$$

and the last rotation to $x'y'z'$ by a matrix B

$$\mathbf{x}' = \mathsf{B}\boldsymbol{\xi}'.$$

Hence the matrix of the complete transformation:

$$\mathbf{x}' = \mathsf{A}\mathbf{x}$$

is the product of the successive matrices:

$$\mathsf{A} = \mathsf{BCD}.$$

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*Unfortunately there is no unanimity in the literature about the definition of the Eulerian angles. The differences are not very great, but often are sufficient to frustrate any easy comparison of the end formulae such as the matrix elements. Greatest confusion, perhaps, arises from the occasional use of left-handed coordinate systems (as by Osgood and Margenau & Murphy). More frequent is the custom of measuring the angle of the line of nodes from the y axis and not the x axis. This practice, which seems characteristic of the British school (cf. Whittaker, Newboult, Ames and Murnaghan) is due to taking the second rotation about η instead of ξ. Our angles ϕ and ψ are then equal to the angles $\phi + \frac{\pi}{2}$ and $\frac{\pi}{2} - \psi$ respectively in such a notation. Continental authors usually follow the definitions given here, except that the meanings of ϕ and ψ are often interchanged. This does not end the confusion, however; many authors of quantum-mechanical discussions appear to use *clockwise* rotations, rather than the counterclockwise convention observed here.