4–4 The Eulerian angles. It has already been noted that since the elements a_{ij} are not independent they are not suitable as generalized coordinates. Before setting up the motion of rigid bodies in the Lagrangian formulation of mechanics it will therefore be necessary to seek three independent parameters specifying the orientation of a rigid body. Only when such generalized coordinates have been found, can one write a Lagrangian for the system and obtain the Lagrangian equations of motion. A number of such sets of parameters have been described in the literature, but the most common and useful are the *Eulerian angles*. We shall therefore define these angles at this point, and show how the elements of the orthogonal transformation matrix can be expressed in terms of them.

One can carry out the transformation from a given cartesian coordinate system to another by means of three successive rotations performed in a specific sequence. The Eulerian angles are then defined as the three successive angles of rotation. The sequence will be started by rotating the initial system of axes, xyz, by an angle ϕ counterclockwise about the z axis, and the resultant coordinate system will be labelled the $\xi\eta\zeta$ axes. In the second stage the intermediate axes, $\xi\eta\zeta$, are rotated about the ξ axis counterclockwise by an angle θ to produce another intermediate set, the

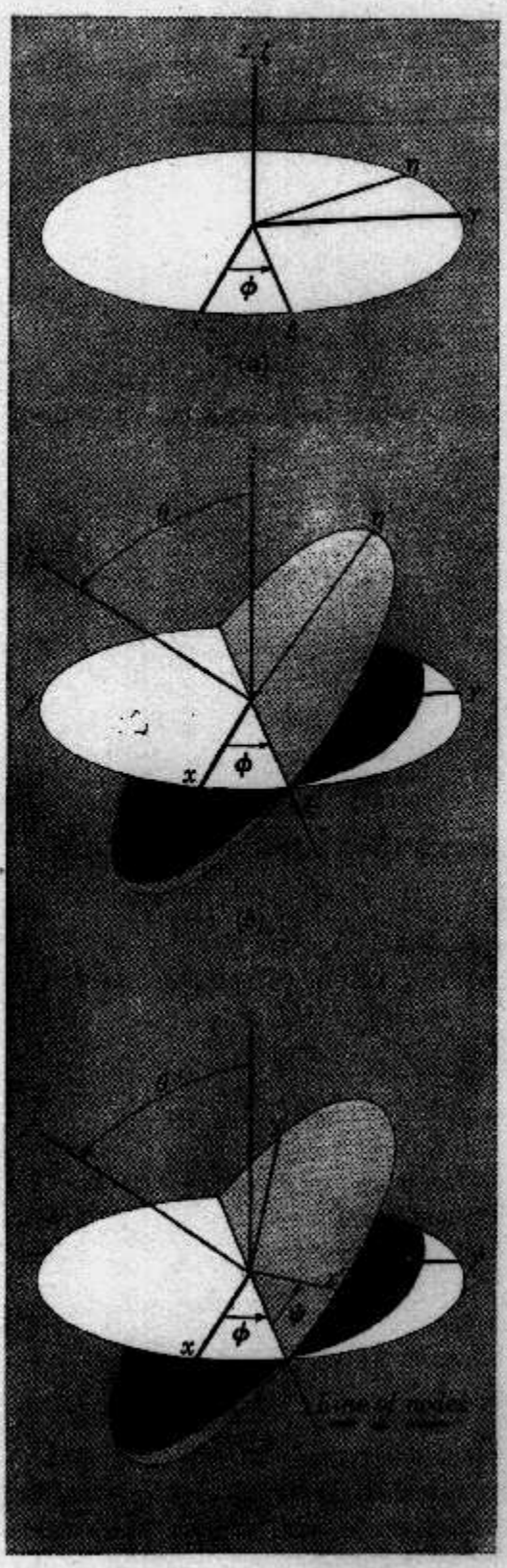

FIG. 4–6. The rotations defining the Eulerian angles.