Using Radio Interferometry to Determine Astrophysical and Geophysical Properties

Timothy Ross
Group Partners- Ben Jeffers, Andrew Halle, Sam Johnson

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Abstract

In this lab we used the radio interferometer located on the roof of Campbell Hall to measure interferometer baselines and source radii. We utilized Fourier transforms to analyze frequency information of our time series data. The best fits for our values were determined by minimizing the sum of squared residuals over a range of test values. Our measured baselines were 15.64m$\pm$.19m east-west and 1.79m$\pm$.09m north-south and the source radii were .27983°$\pm$.00047°for the Sun, and .2555°$\pm$.0027°for the Moon.

1 Introduction

Radio interferometry is useful for many fields, not just astronomy. The measured quantities depend on multiple variables, which in different fields can be determined very accurately. For geophysics, interferometry of a source of well known position yields accurate real-time distances between interferometer antenna. We used a known object, the Sun, Moon, and Cassiopeia A, to determine our interferometer baseline accurately. In addition, the angular radii of sources can be found accurately from the modulation of the signal. We found the radii of the Sun and Moon using this method.

2 Observations

2.1 The Interferometer

In this lab we used the radio interferometer, whose schematic is shown in figure 1. There are two antenna, which measure electric field in the form of voltage, and that voltage signal is passed through a series of band

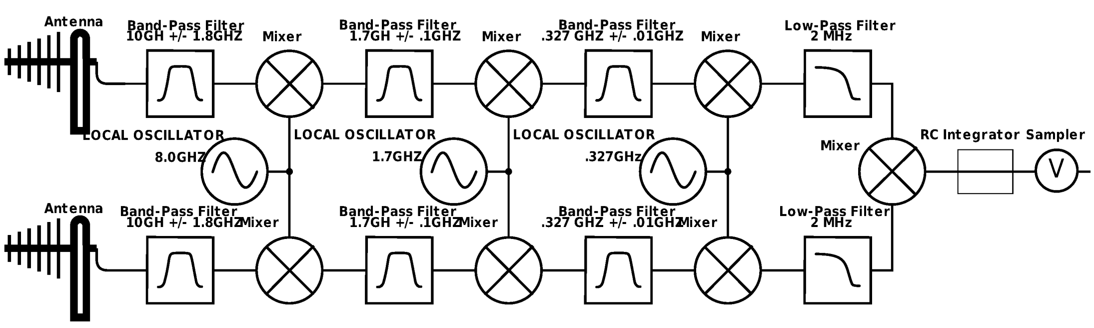

Figure 1: Simplified schematic of the interferometer used in the lab. The signal measured at the two antenna are mixed and filtered individually, selecting a 2MHz range of frequencies around 10.027GHz. The two signals are then mixed together, time integrated, and sampled.

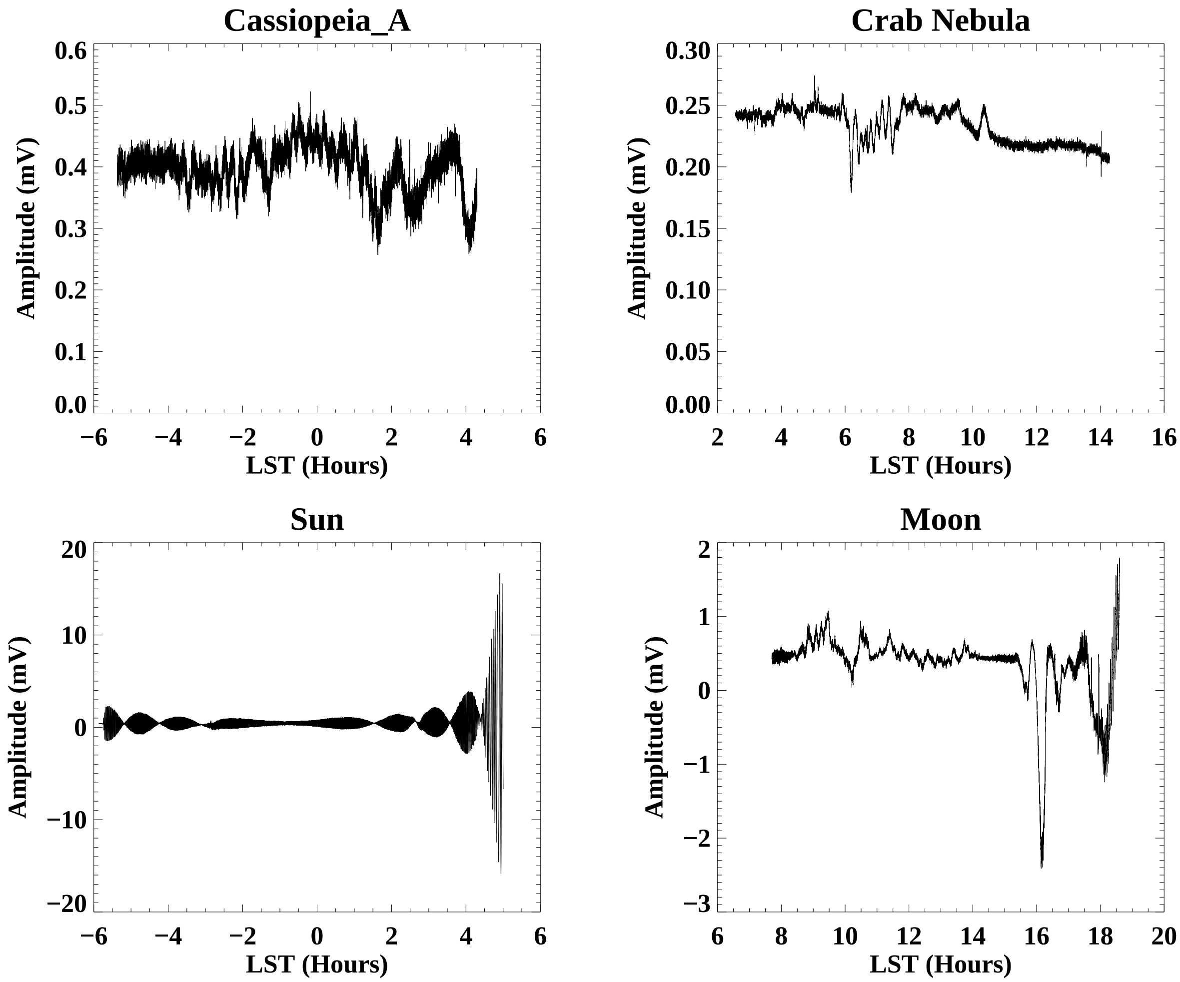

Figure 2: The time series data of four sources, Cassiopeia A, the Crab Nebula, the Sun, and the Moon. They are plotted measured amplitude against local sidereal time (LST).

pass filters and mixers which serve to reduce the signal from centered around 10.027GHz to a range from 2MHz to 0Hz. This signal is mixed with the signal from the other antenna, which has undergone the same transformation. The signal is then time averaged, and the average is recorded by the sampler. What we are recording is the interference between the two signals, caused by the path length difference traveled by the signal. That difference changes with time, as the telescope follows a source. There are many different mathematical interpretations for this information; not only is it the path length difference as a function of time, but it can bee seen as sampling the sky in frequency or Fourier space. This interpretation is especially useful, and will be used in section 3.4. In order to follow the sources, the lab has a procedure named `follow` that takes an input right ascension and declination, and points at and follows the position as the earth moves. For the Sun and Moon, `follow` has special keywords that point to and follow the Sun and Moon, which have time varying right ascension and declination. The program also records the position of the Sun and the Moon as it is follow them. The program `startchart1` is used to record data from the interferometer, which records the amplitude and the local sidereal time (LST) for each measurement, which can be converted to hour angle from the right ascension.

2.2 Point Sources

We gathered time series data from the interferometer of point source Cassiopeia A. These data are shown in figure 2. Due to concern that signal to noise was too low for this source, we also collected data of the Crab Nebula, also shown in figure 2.

2.3 Continuum Sources

We gathered time series data from interferometer of the Sun and Moon, two continuum sources. These data are shown in 2. The Sun data have very high signal to noise; both the fringe and modulation are clearly visible without filtering.

3 Data Analysis

3.1 Fourier Analysis

The time series data contain a large amount of information in frequency space, which can be obtained from a Fourier transform. Not all of the data taken were usable because of pointing limits or occulting buildings, and the ranges of the data used are shown in table 1. The frequency space data are shown in figure 3 for all four sources. In the point sources the fringe frequency is given by equation 14 in the lab manual, which is $f_f = \left[\frac{B_{ew}}{\lambda}\cos\delta\right]\cos h_{s,0} - \left[\frac{B_{ns}}{\lambda}\sin L\cos\delta\right]\sin h_{s,0}$. Assuming that $B_{ew} >> B_{ns}$, and that $\cos\delta$ is of order unity, at zenith these terms for a baseline of around 20 m and wavelength 3 cm, are about 600 cycles per radian. Converting to cycles per second knowing that there are $24 \times 60 \times 60$ seconds in 2π radians, the peak is at 40 mHz, which is the order of magnitude of where we see peaks in all four Fourier transforms. We know now from this equation that this is the important information encoded in these data, so we can filter out the other frequencies, which are just noise. Anything past the maximum fringe frequency is not contributed by the fringe, and can be zeroed. This break can be clearly seen in the Sun data, where there is an abrupt drop in power after the maximum fringe frequency. Zeroing all frequencies after 40 mHz, and also zeroing the dc contribution near zero, we inverse Fourier transform and get filtered data; an example is shown in figure 4 for the Moon. Before filtering the modulation due to being a continuum source is not visible, but after filtering it is easy to see.

Table 1: Ranges of the usable data

Source	Begin (HA)	End (HA)
Cassiopeia A	-4	4
Crab Nebula	-3	3
Sun	-5.2	6
Moon	-4	4

3.2 Baselines from Least-Squares Analysis

The amplitude of the point sources can be expressed as a linear combination of two terms that are known functions of time, Q_{ew} and Q_{ns}. That equation is

$$F(h_s) = A\cos(2\pi\Omega) + B\sin(2\pi\Omega) \tag{1}$$

$$\Omega(Q_{ew}, Q_{ns}, h_s) = Q_{ew}\sin h_s + Q_{ns}\cos h_s \tag{2}$$

$$Q_{ew} = \left[\frac{B_{ew}}{\lambda}\cos\delta\right] \qquad Q_{ns} = \left[\frac{B_{ns}}{\lambda}\sin L\cos\delta\right] \tag{3}$$

We can use least squares to determine the coefficients in equation 1, if we assume a value of Q_{ew} and Q_{ns}. From the best fit coefficients, we can find the residuals, as described in the least squares fitting handout.

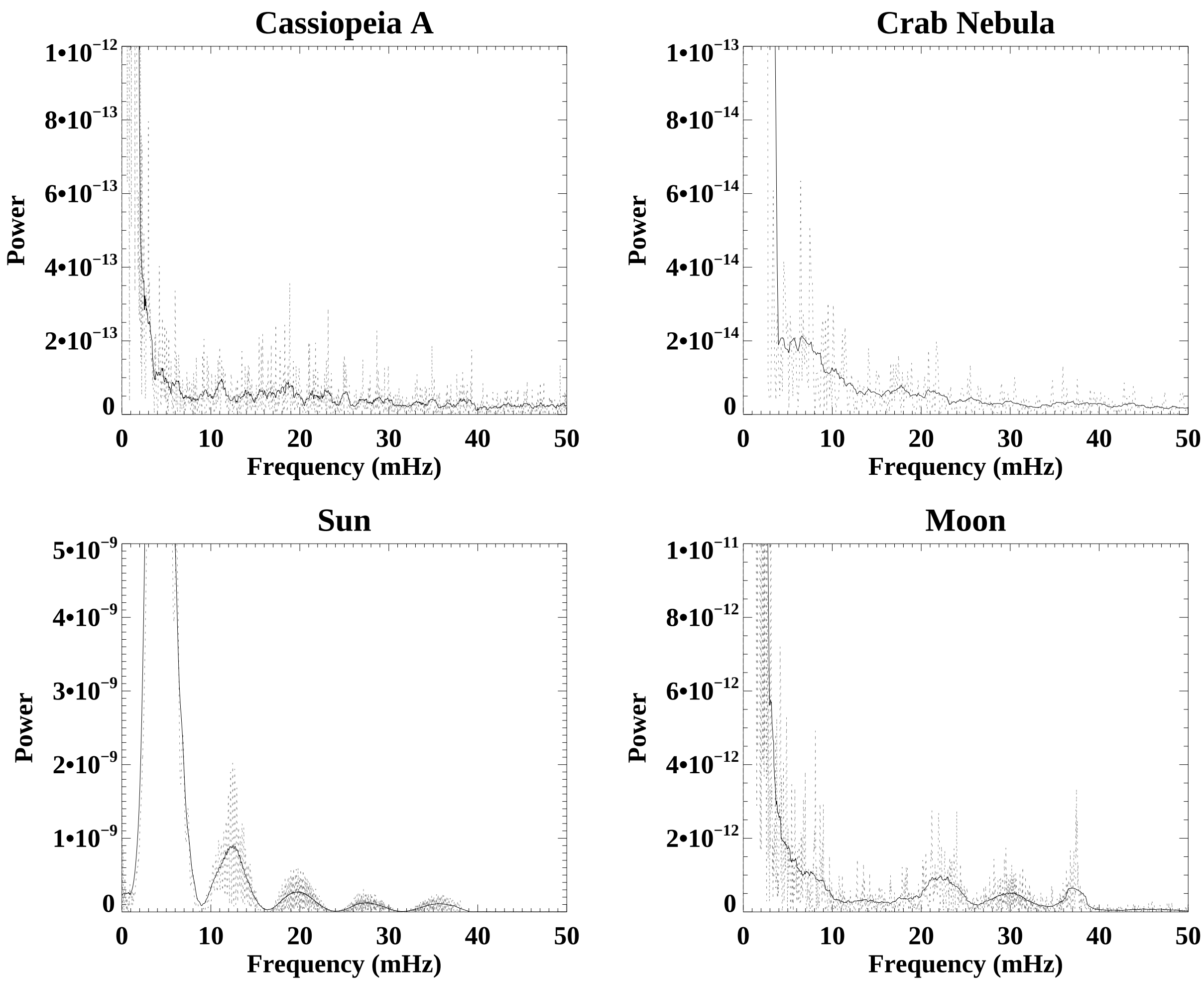

Figure 3: The time series data of four sources, transformed into frequency space via Fourier transforms. The entire data range were not used for each transform, since some of the time the sources were out of range or blocked. The point sources both have peaks around 20 mHz, where the fringe frequency is at maximum. The continuum sources have a modulating envelope clearly visible, due to their extended nature. The measured data are plotted in gray, and smoothed data overplotted in black.

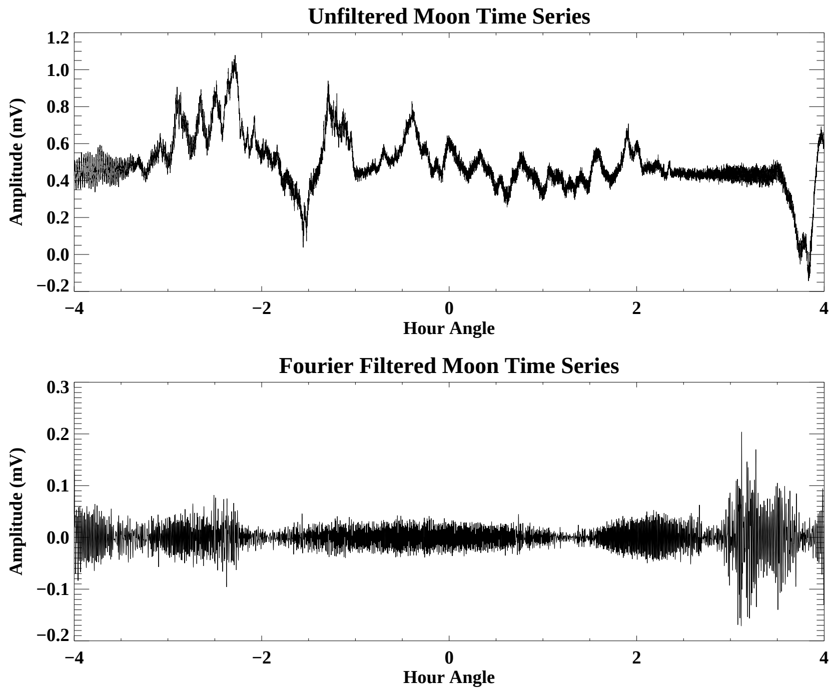

Figure 4: The Moon time series was Fourier transformed, and then all frequencies not between .01 Hz and .04 Hz were zeroed. The filtered Fourier data were inverse Fourier transformed into real space. This removed much noise, the modulating function of the continuum source is now clearly visible.

Table 2: Baseline fits

Source	B_{ew}	B_{ns}	Covariance
Cassiopeia A	15.64m±.19m	1.79m±.09m	.201
Sun	15.619m±.062m	1.624m±.011m	.321
Moon	15.59m±.11m	1.661m±.059m	-.807

From the sum of squared residuals, we can compare the fit of different Q values. We know the declination of our source, and we know the frequency of the signal and thus the λ. Therefore, if we find the best fit Q values, we know, the best fit baseline. This fitting is done first assuming the baseline north-south (Q_{ns}) is zero, and iterating over the value of Q_{ew}. This is done for both Cassiopeia A, and for the Sun data around zenith, shown in figure 5 and 6 respectively. The high signal to noise of the Sun data are apparent, as the minimum is very near zero, while in Cassiopeia A the minimum is much farther from zero. relative to the median value.

There are two dimension to the baseline of the interferometer, north-south and east-west. Thus, an array of baseline values in both dimension must be taken to find the global minimum. This is done by picking a Q_{ew} value, and then calculating a range of Q_{ns} values, and doing this for a range of Q_{ew}. This is two nested loops, and produces a two dimensional array of sum of squared residuals. The array for Cassiopeia is shown in figure 7, the Sun in figure 8, and the Moon in figure 9. The minimum values are shown in table 2. The variances were calculated using the curvature matrix found by Taylor expanding around the minimum. The Taylor expansion around the minimum gives the following formula for the sum of squared residuals:

$$\Delta S^2 = \frac{1}{2}\frac{\partial^2 S^2}{\partial Q_{ew}^2}\Delta Q_{ew}^2 + \frac{\partial^2 S^2}{\partial Q_{ew}\partial Q_{ns}}\Delta Q_{ew}\Delta Q_{ns} + \frac{1}{2}\frac{\partial^2 S^2}{\partial Q_{ns}^2}\Delta Q_{ns}^2 \tag{4}$$

This gives the following curvature matrix:

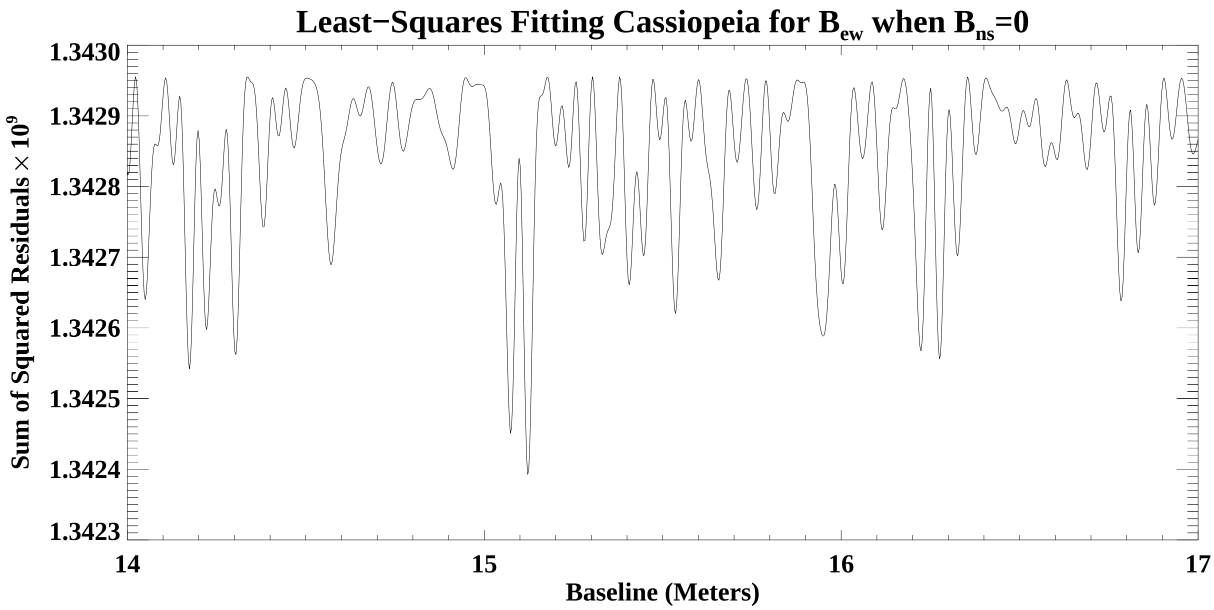

Figure 5: The sum of squared residuals for fitting B_{ew} to Cassiopeia A, with B_{ns} held at 0. Lambda is 3 cm in this evaluation, determined from the schematic in figure 1. The minimum shown here is zoomed for clarity, the residuals are very nearly constant for all guess B_{ew} values, due to low signal to noise.

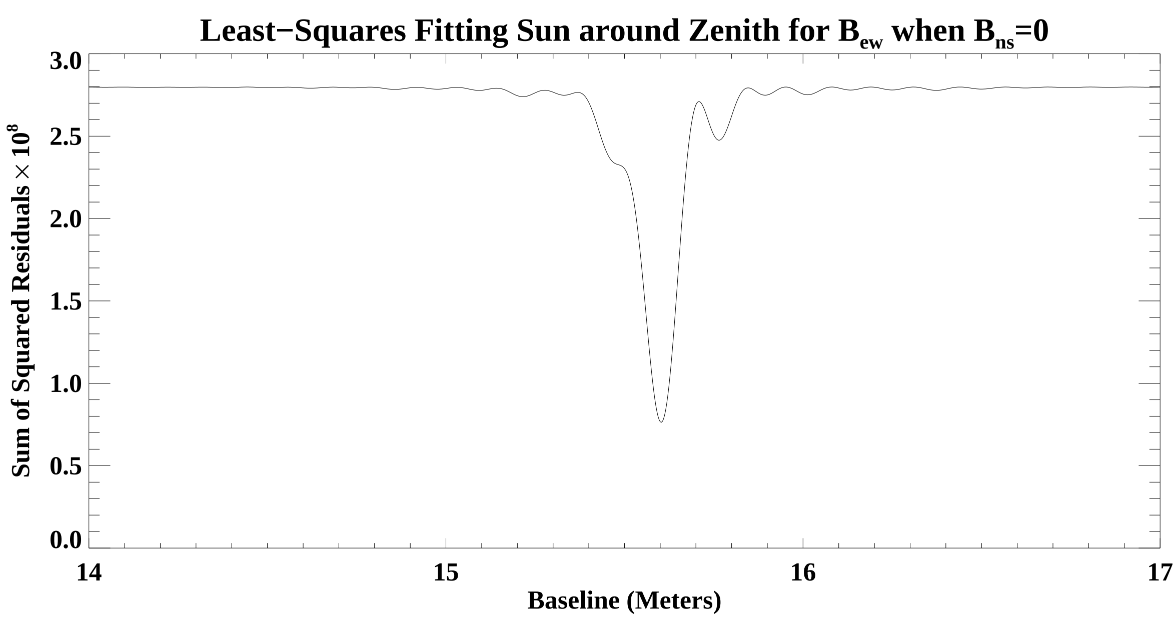

Figure 6: Demonstration of the fitting code with high signal to noise data, in this case the sun data. The global minimum is deeper than any other local minimum, and the residuals vary over a large range.

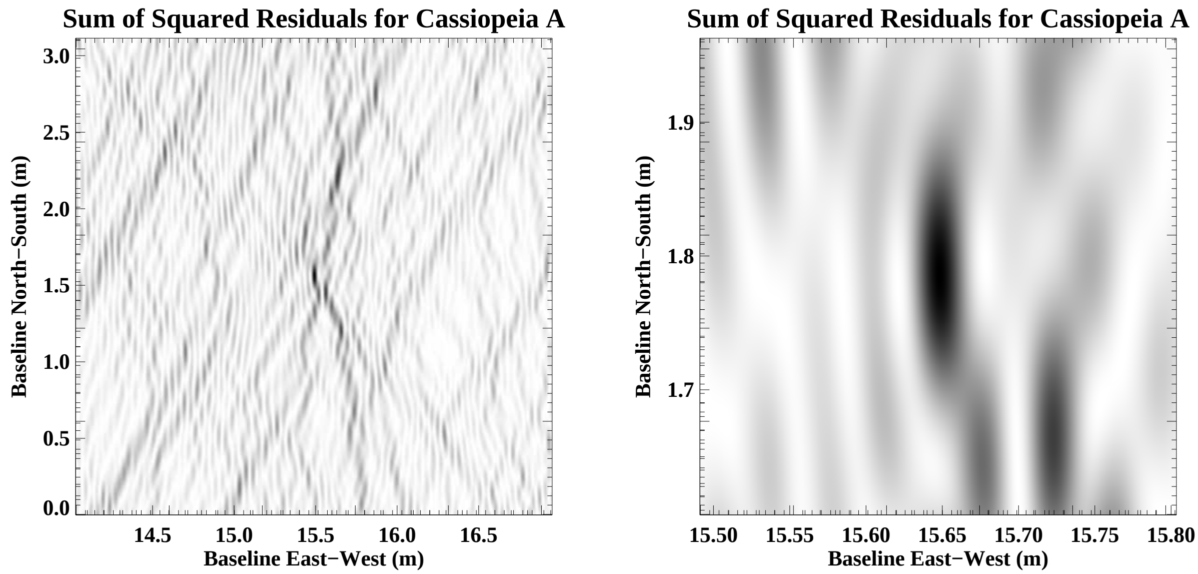

Figure 7: The two dimensional array of sum of squares residuals found from least squares fitting various values of B_{ew} and B_{ns} for Cassiopeia A. The darker color indicates smaller value. The right image is zoomed ten times on the minimum. The low signal to noise is apparent in the tubulent scattering of the minima

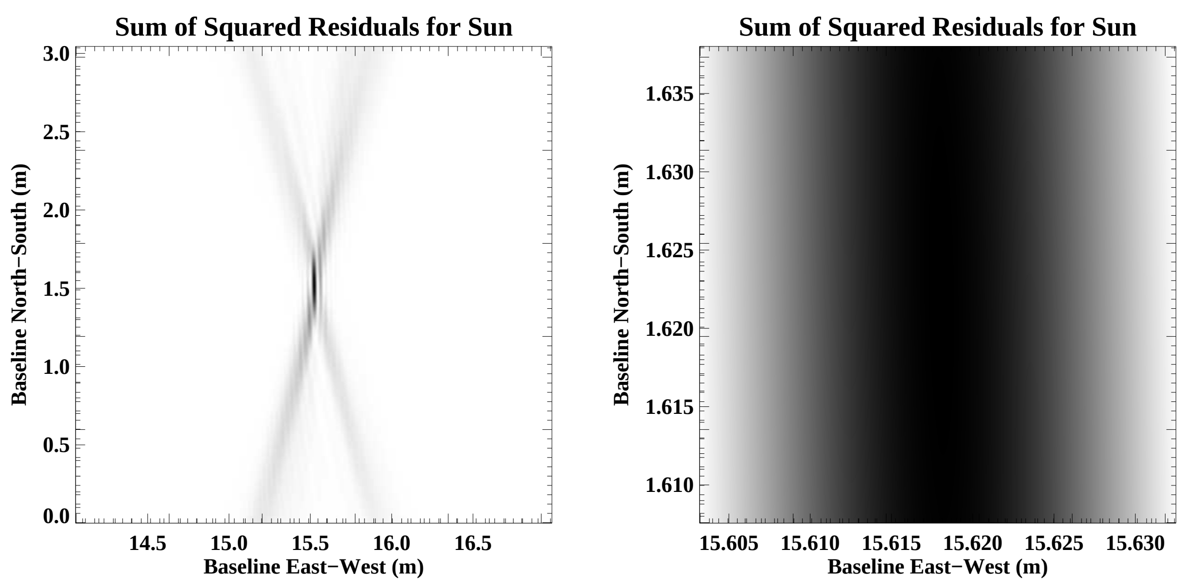

Figure 8: The two dimensional array of sum of squares residuals found from least squares fitting various values of B_{ew} and B_{ns} the Sun. The darker color indicates smaller value. The right image is zoomed one hundred times on the minimum. The high signal to noise is apparent in the residuals, with a singular well defined minimum.

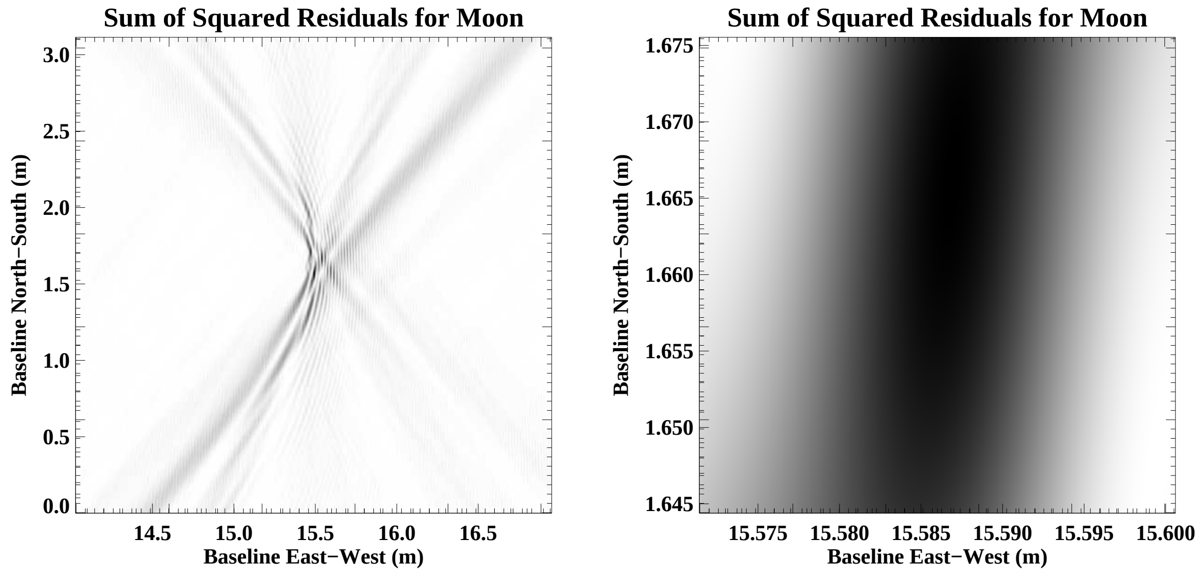

Figure 9: Two dimensional array of sum of squares residuals found from least squares fitting various values of B_{ew} and B_{ns} the Moon. The darker color indicates smaller value. The right image is zoomed one hundred times on the minimum. The decent signal to noise can be seen in the residuals. There are a group of minima and a diffraction like pattern, but they are well grouped, unlike Cassiopeia A.

$$[\alpha] = \begin{bmatrix} \frac{1}{2}\frac{\partial^2 S^2}{\partial Q_{ew}^2} & \frac{\partial^2 S^2}{\partial Q_{ew}\partial Q_{ns}} \\ \\ \frac{\partial^2 S^2}{\partial Q_{ew}\partial Q_{ns}} & \frac{1}{2}\frac{\partial^2 S^2}{\partial Q_{ns}^2} \end{bmatrix} \tag{5}$$

The variances are the diagonal elements of the inverse of the curvature matrix, and the covariances are the off-diagonal elements. The square root of the variance is used to give the error of the value. The error corresponds to the expected range of the standard deviation of a random variable. For our purposes, the variance tells us that if we ran the same experiment many times, the error is the range over which we would expect the found value to occur. Since we only have one experiment, we say that the true value is within this range of our found value. The variances of the two coefficients depend on each other, by a measured called the covariance. The normalized covariance is given by the formula, where XXI is the inverse of the curvature matrix:

$$ncov_{ik} = \frac{XXI_{ik}}{\sqrt{XXI_{ii}\ XXI_{kk}}}\ . \tag{6}$$

3.3 Source Diameter from Real Space

The Sun and Moon are continuum sources. The projected sine wave of the interferometer has a wavelength that is significant compared to the angular size of the Sun and Moon, and thus the received power is an integral over the area of the Sun and Moon. The different parts of the continuum source interfere with each other, and this interference cause the Sun and Moon to look like a point source, modulated by some modulating function, which is a function of radius and fringe frequency. The modulating function depends on the intensity distribution of the source, and if we assume uniformly bright circular Sun and Moon, the modulating function, as derived in the lab manual, can be calculated analytically with the function:

$$MF_{theory} \approx \delta h \sum_{n=-N}^{n=+N} \left[1 - \left(\frac{n}{N}\right)^2\right]^{1/2} \cos\left(\frac{2\pi f_f R n}{N}\right) \tag{7}$$

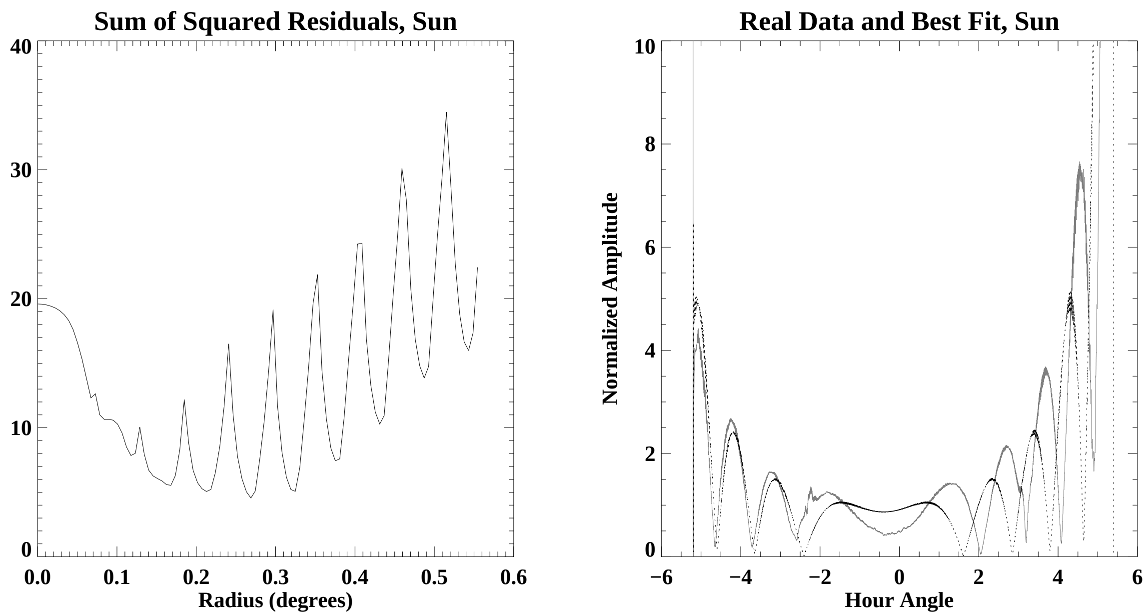

Figure 10: On the left, the sum of squared residuals comparing the Sun time series and the modulating function for various radii. On the right, the Sun time series in gray and the best fit modulating function in black. The global minimum is only slightly deeper than other minima, and the fit is inexact.

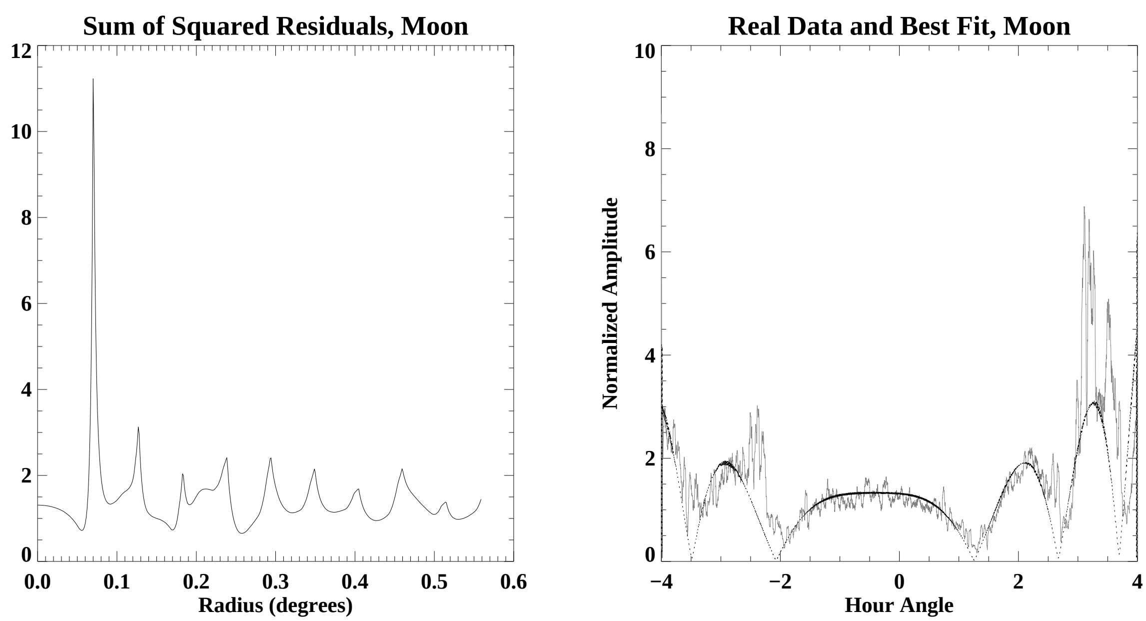

Figure 11: On the left, the sum of squared residuals comparing the Moon time series and the modulating function for various radii. On the right, the Moon time series in gray and the best fit modulating function in black.

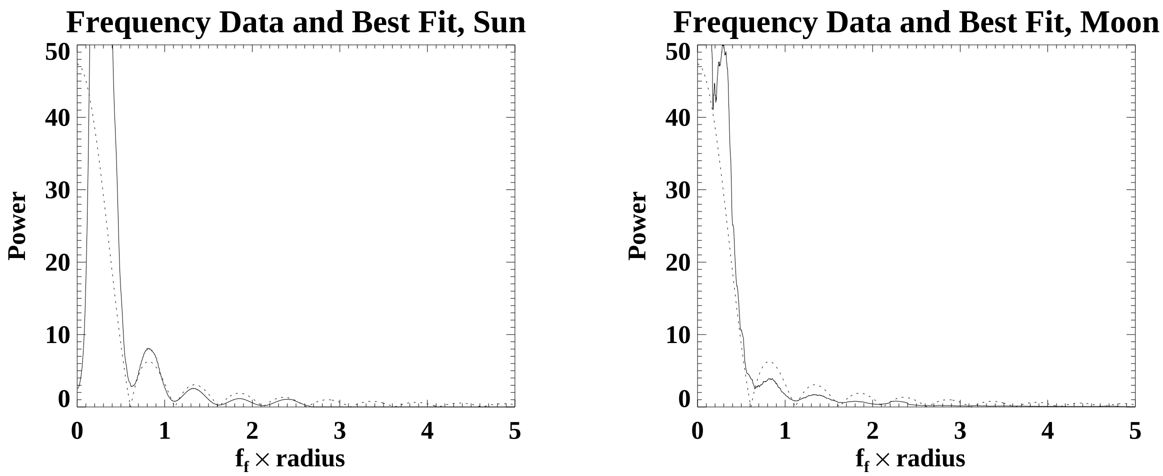

Figure 12: Fourier transform of the Sun and Moon data, and the best fit modulating function. Only the first 5 zeros are used, since the next zeros occur past the maximum fringe frequency.

The modulating function is a function of just fringe frequency and radius. Fringe frequency can be expressed as a function of just time once Q is known, so the Modulating function can be expressed as a function of time, and compared to our time series. With `envelope_least_squares` I calculated the modulating function for a given radius, then multiplied it by the best fit point source function found with `least_squares` to generate theoretical time series data. These theoretical data were compared to the real time series data. There was difficulty initially because the fitting procedure is not accurate enough over the full range of the time series, since the modulating function introduces 180°phase shifts to parts of the data. To compensate, the two data sets were root mean square averaged over a range of 64 data points. These two averaged functions were subtracted, squared, totaled and then square rooted to obtain the sum of squared residuals for that particular radius value. This process was repeated for a range of radius values, and the sum of squared residuals are shown in figure 10 for the Sun, and figure 11 for the Moon, along with the best fit. The best fit radii are shown in table 3. The variances were calculated by obtaining the curvature matrix by Taylor expansion around the minimum.

Table 3: Radius fitting Continuum Sources

Source	Real Space	Fourier Space
Sun	$.269° \pm .02°$	$.27983° \pm .00047°$
Moon	$.257° \pm .02°$	$.2555° \pm .0027°$

3.4 Source Diameter from Fourier Space

The fit for the modulating function is not very good, and there are issues with the found baseline of 15.5 meters, since we know that the baseline is actually 17 meters, a discrepancy which will be addressed in section 4.1. The fit for the source diameter depends on the value of the baseline, so an incorrect baseline would lead to an incorrect radius. However, it is possible to deduce the radius without needing the baseline, because the modulating function depends on the combined fringe frequency times radius, and the data after a Fourier transform are in frequency space. We can use the envelope of the Fourier transform, and compare it to the envelope of the modulating function in frequency space. Both envelopes have distinct zeros. The zeros of the theoretical modulating function, which has no noise, is found by finding the indexes where the function goes from positive to negative. For the data in frequency space, the data are root mean square smoothed, and then the data scanned, and all minima over the range of 128 data points are saved as the zeros of the Fourier transform. Only the first 4 zeros are good, because after that the zeros occur past the maximum fringe frequency. We now have two sets of zeros, one occuring at specific $f_f \times R$ values, and one occuring at specific f_f values. The following matrix equation can be constructed, and least-squares fitted:

$$\begin{bmatrix} (f_f)_{observed} \\ (f_f)_{observed} \\ \vdots \end{bmatrix} \begin{bmatrix} R \end{bmatrix} = \begin{bmatrix} (f_f \times R)_{theory} \\ (f_f \times R)_{theory} \\ \vdots \end{bmatrix} \tag{8}$$

This matrix solving method allows for the finding of variances through the curvature matrix, and the results and errors are shown in table 3.

4 Interpretation

4.1 Baseline Measurements

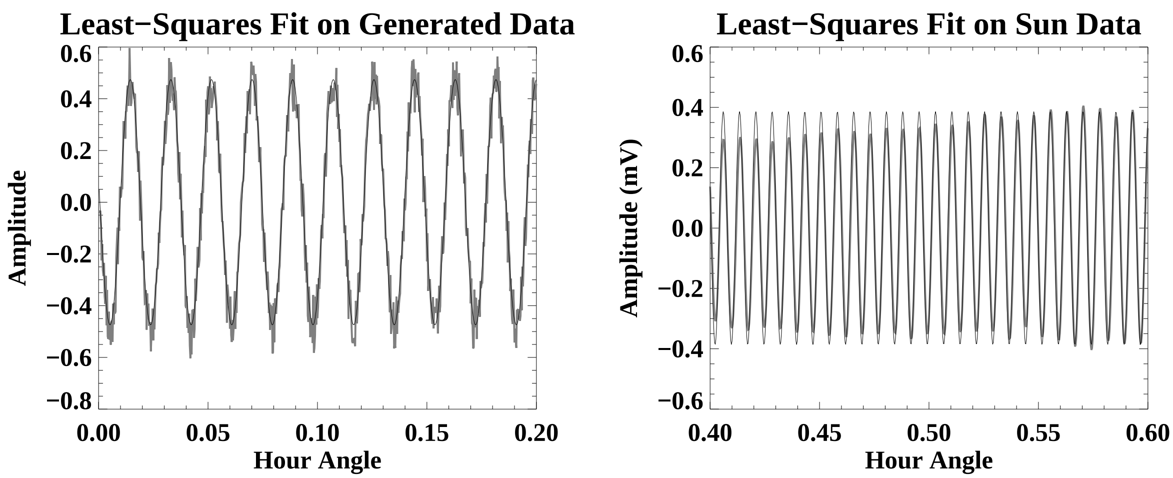

Figure 13: The strength of fit for generated data using the equation for fringe, random position, and random coeffients on the left. On the right is the strength of fit for Sun data around zenith. The gray are the data, and the black are the best fit.

The length of the baseline is problematic. All of the least-squares fitting of the fringes yielded a result of about 15.5 m, but the actual distance is about 17 m, as told by the professor. Much care was taken to reduce all possible error in the least squares fitting, and the most perplexing element was the strength of the fit. The procedure was tested on both the Sun data, which have very high signal to noise, and with generated data using the given equation for fringe frequency. Shown in figure 13 is the best fit calculated by the procedure, both for generated data, and for the Sun data. The strength of the fit is very high, as shown visually in the fit. The strength of the fit implies that the found Q is correct. However, the Q found is too low, and the two possible other sources of error are the declination and the wavelength. For the Sun, the declination is near zero, and its contribution is minimal, so it is unlikely that is the source of error. The wavelength would need to be 10% greater to account for the error, but 3 cm is found directly from the setup. It is possible that the hour angles of the Sun data are not properly interpolated, because `follow` records LST less frequently than `startchart1`, but the analysis of the Cassiopeia A data also has a minimum at 15.5 m, which has LST recorded by `startchart1`. I was unable to determine the source of this error in the calculation, and am at a loss of more places to look for error.

This error is real, not just on faith that the baseline is 17 m. The continuum source calculation in real space use the baseline value, and substituting in 17 m give a better fit, shown in figure 14. The global minimum is both more distinct from the other mimima, and the calculated variance is half the variance calculated with 15.5 m. The calculations of fringe frequency and modulating functions are independent of the calculations

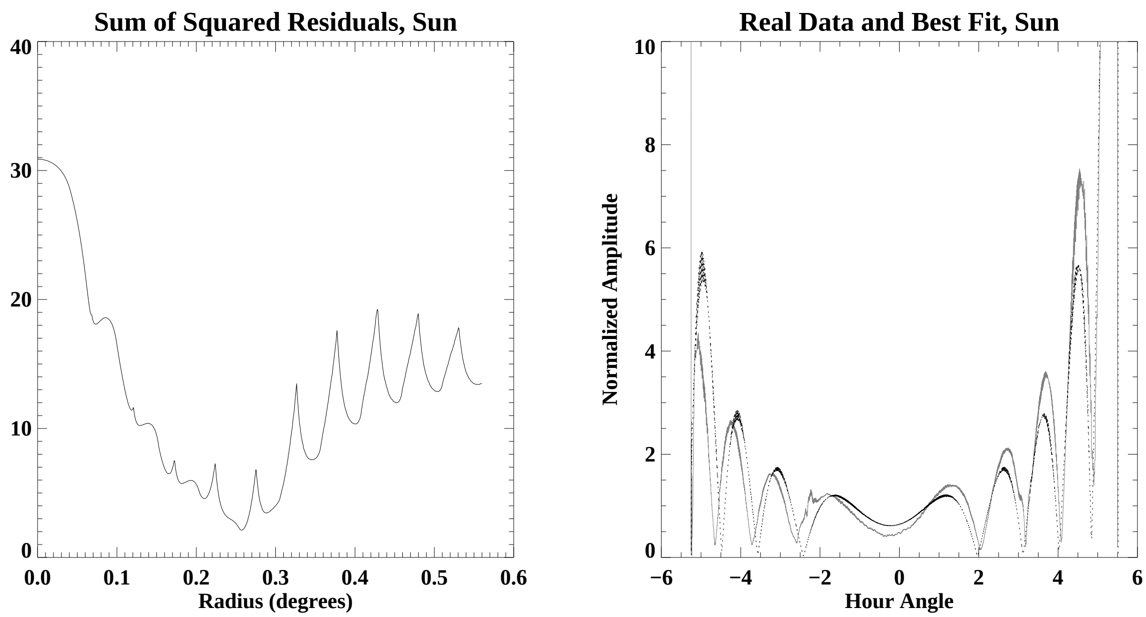

Figure 14: On the left, the sum of squared residuals comparing the Sun time series and the modulating function for various radii, assuming $B_{ew} = 17$ m. On the right, the Sun time series in gray and the best fit modulating function in black. The global minimum is much deeper than all local minimum, and the best fit appears better.

in determining the baseline, and free of that calculation's error. For this reason I calculated the continuum source radii in Fourier space as well as real space, because the Fourier space calculation is independent of the baseline calculation altogether. The results of the Fourier space calculation have a smaller error, and are within the error of the real space. The baseline measurements have systematic error of an unknown source, but I was able to isolate that error to that calculation alone, and derive the other quantities independently and accurately.

5 Group Contributions

Most of the work of this lab was in the analysis and coding, which was all done individually. Andrew Halle took the data for Cassiopeia A, the Sun, and the Moon, I took data for the Crab Nebula

6 Conclusion

Interferometry is a tool for astrophysics, and a tool for other fields like geophysics. We were able to determine the distance between two telescopes accurately for a known point source, though there was some issues with final result, as it contrasted with the known value, with a discrepancy outside of the calculated error. However, we were able to independently derive the radii of the Sun and Moon, without the error from the baseline measurement. Solving in frequency space, we used the Fourier transform of the data to determine the radii without needing to know the fringe frequency as a function of hour angle. Solving in real space, we used the found baseline values to calculate the radii as a proof of concept. Our measured baselines were 15.64m$\pm$.19m east-west and 1.79m$\pm$.09m north-south and the source radii were .27983°$\pm$.00047°for the Sun, and .2555°$\pm$.0027°for the Moon. The tools we learned are used by larger and more precise arrays and telescopes to calculate the movement of tectonic plates, or the sizes even smaller astrophysical sources. The skills of analyzing the frequency domain, least squares fitting over multiple parameters, and calculating errors are skills that we will continue to use in both our astrophysics careers, or any scientific feild we enter.

7 Software

All the programs used in this lab are in /home/willieross/Astro121/week6/ and there are several necessary data files, which are also located in that directory, cas_a_ra_dec.dat cas_a_data.sav crab_ra_dec.dat crab_nebula_data.dat sun_all_day_data.dat sun_all_day_follow.dat moon_all_night_follow.dat moon_all_night.dat. The programs used in the lab are listed below with their documentation:

```
----- Documentation for ./generate_powerspec.pro -----
 NAME:
       generate_powerspec

 PURPOSE:
       loads data file, arranges hour angle in correct order,
       shows power spectrum and the results of fourier filtering

 EXPLANATION:
       The data are loaded and hour angle is made continuous.
       The user is prompted foring sectioning the data
       to allow removal of bad data, and then the data are
       fourier transformed, smoothed, and then filtered,
       removing areas away from the fringe frequency, and then
       converted back to real space

 CALLING SEQUENCE:
       generate_powerspec, name, ha, tseries,fft,f

 INPUTS:
       name: name of the oject

 OPTIONAL INPUTS:

 KEYWORD PARAMETERS:

 OUTPUTS:
       tseries: output time series, after filtering and shortening
       ha: hour angle of the filtered time series

 OPTIONAL OUTUTS:
       fft: value of fourier transform
       f: x values for fft
 EXAMPLES:
       generate_powerspec, 'sun', ha, tseries

 REVISION HISTORY:
       written and documented 3/26 by Timothy Ross



----- Documentation for ./least_squares.pro -----
 NAME:
       least_squares_baseline_main

 PURPOSE:
       least squares fits the interferometer data to find
       the best fit baseline
```

```
EXPLANATION:
      Loads the data choosen by the user. Those data are filtered
      if desired. A first degree polynomial is fitted and removed
      data, zeroing it. From the guessed baselines, and input size
      and scale parameters, an array of Q valuiies to test is
      generated. The data are then least-squares fitted by matrix
      methods to determine the residuals for each Q value. The
      residuals are formed into a grid for display purposes, and
      the minimum is subtracted, and then displayed with the axes
      displaying the baseline values. The best fit baseline is
      calculated from the Q at minimum residual and diplayed. The
      smallest residual value is found and then the fit from that
      residual is recalculated. If desired, that results are saved

CALLING SEQUENCE:

      least_squares_baseline_main, size, name,image,b_x,s_y,$
      s_min,t_prime,tseries,ha_radians,dec_radians, short=short,$
      scale_mult=scale_mult, save=save,fourier=fourier,$
      baseline=baseline

INPUTS:
      size: 2 element array for x size and y size
      name: which data to call. options are 'cas_1', 'crab',
            'sun', 'moon', 'fake'
OPTIONAL INPUTS
      none

KEYWORD PARAMETERS
      short: if set, uses only a range of the data
      save: if set, the image is saved to file
      scale_mult: how far around the initall guess to check
      fourier: if set, fourier filters the data. It is transformed
               to fourier space, a section in the range
               [-outside,-inside] and [inside,outside] is
               transformed back into real space
      baseline: The initial baseline value to test around


OUTPUTS:
      image: 2D image of the values
      b_x: slice of the east-west baseline at minimum
      s_y: values of the slice
      s_min: minimum value of the residual
      t_prime: reconstructed best fit
OPTIONAL OUTUTS:

EXAMPLES:
      least_squares_baseline_main, [128,128],'fake',$
      image,b_x,s_y,s_min,t_prime,short=[-5,5],save='test.dat'

REVISION HISTORY:
      written and documented 3/26 by Timothy Ross
```

```
----- Documentation for ./covariance.pro -----
 NAME:
       covariance

 PURPOSE:
       Takes a 2d array of residuals and finds the
       variance of the derived coeffiencts, and
       their associated covariance

 EXPLANATION:
       loads either a full save file with image, residuals
       and grid size, or takes them as input, and takes
       the three second derivatives, and uses them to
       calculate the curvature matrix, and then the
       variance and covariance

 CALLING SEQUENCE:
       covariance, image, s_min, d_ew,d_ns, save=save, var_vec, ncov

 INPUTS:
       image: the 2d array of sum of squares residuals
       s_min: the value of the minimum
       d_ew: grid scale of q_ew
       d_ns: grid scale of q_ns

 OPTIONAL INPUTS
       save: save file including all the inputs

 KEYWORD PARAMETERS

 OUTPUTS:
       var_ver: vector containing variances
       ncov: matrix containing covariances

 OPTIONAL OUTUTS:

 EXAMPLES:
       covariance, image, s_min,d_ew,d_ns,save='cas_1_64_64.sav'
       finds the covariance of the fit from the file

 REVISION HISTORY:
       written and documented 3/26 by Timothy Ross



----- Documentation for ./envelope_least_squares.pro -----
 NAME:

       envelope_least_squares

 PURPOSE:
```

```
      takes time series data, uses the baselines found earlier,
      and calculates the modulating function, and fits it with
      least squares of a range of radii to determine best fit
      radii, also giving variance on the radius

 EXPLANATION:
      The fringe frequency is calculated from the derived baseline
      and used to calculate the modulating function for a range of
      radii and the found modulating function is multiplied by the
      best fit point source function, to give a theoritical function.
      The theoritical and observed functions are squared and averaged
      to get their envelopes, and then the residuals between the two
      smoothed functions is found for each radius value, to dtermine
      the best fit radius. The variance of the best fit value is found
      from the derivatives around the minimum.

 CALLING SEQUENCE:
      pro envelope_least_squares, tseries, ha_radians, dec_radians,$
        t_prime, baseline, radius,variance=variance, size=size,$
        scale=scale,radius_initial=radius_initial ,output=output

 INPUTS:
      tseries: time series data of sun or moon
      ha_radians: radian values of hour angle for each tseries value
      dec_radians: array of declinations, same length as tseries, in
      radians
      t_prime: best fit point-source function from equation 12
      baseline: values of baseline, [B_ew,B_ns]

 OPTIONAL INPUTS:

 KEYWORD PARAMETERS:
      size: the amount of values to iterate over, default 512
      scale: the range searched over, from r-r*scale to r+r*scale,
             default 1
      radius_initial: the value of radius to search around, default
                      .28 degrees
      output: string containing ps filename to output best fit plot

 OUTPUTS:
      radius: the best fit radius of the object, in degrees

 OPTIONAL OUTUTS:
      variance: the variance of the found radius

 EXAMPLES:
      envelope_least_squares, tseries, ha_radians, dec_radians, $
      t_prime, baseline, radius,variance=variance


 REVISION HISTORY:
      written and documented 4/2 by Timothy Ross

----- Documentation for ./envelope.pro -----
```

```
NAME:

      envelope

PURPOSE:
      takes time series data of the sun or moon, and from
      the modulating envelope in fourier space, determines
      the angular diameter

EXPLANATION:
      The data are transformed to frequency space via Fourier
      Transform, where the modulating envelope is a function of
      frequency. The modulationg function is calculated numerically
      in terms of fringe frequency times radius. The mun data exist
      in fringe frequency space, and so after smoothing, zeros/minima
      are found in terms of fringe frequency. Least squares are used
      to determine the best fit R that reduces the sum of square
      residuals between the zeros of the modulating function and
      the zeros of the data. The curvature matrix from the least
      squares is used to get the variance of the radius

CALLING SEQUENCE:
      envelope, name, radius, variance=variance

INPUTS:
      image: the source to analyze, either 'sun' or 'moon'

OPTIONAL INPUTS:

KEYWORD PARAMETERS:

OUTPUTS:
      radius: the best fit radius of the object, in degrees

OPTIONAL OUTUTS:
      variance: the variance of the fit

EXAMPLES:
      envelope, 'sun', radius, variance=variance
      finds the radius of the sun

REVISION HISTORY:
      written and documented 3/26 by Timothy Ross



----- Documentation for ./fake_data.pro -----
NAME:
      fake_data

PURPOSE:
      creates fake time series data from the equation 12
      in the lab manual. The data have noise and random ra, dec,
      and are saved to the files 'fake_data.dat' and
```

```
      'fake_ra_dec.dat'

EXPLANATION:
      Data is generated using the equation 12, then saved

CALLING SEQUENCE:
      fake_data, b_ew, b_ns

INPUTS:
      b_ew: the east-west baseline of the data
      b_ns: the north-south baseline of the data

OPTIONAL INPUTS:

KEYWORD PARAMETERS:

OUTPUTS:

OPTIONAL OUTUTS:

EXAMPLES:
      fake_data, 15.5, 1.7
            generates data that a source at random ra and dec
            would produce for a inferometer of baselines 15.5
            and 1.7

REVISION HISTORY:
      written and documented 3/26 by Timothy Ross
```