

ASTRONOMY WITH THE 21-CM LINE; BENCH WORK WITH COAX CABLES, WAVEGUIDES, AND IMPEDANCE

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1. GOALS

- Measure the 21-cm line power spectrum from atomic Hydrogen in the Milky Way.
- Reduce data by averaging, medianing.
- Remove the instrumental contribution to the spectrum.
- Use rotation matrices to convert among various spherical coordinate systems.
- Learn about reflections and minimizing them with impedance matching.
- Derive propagation velocities by measuring the wavelength of standing waves.
- Measure cable lengths by measuring reflections.
- Empirically explore waveguides and their cutoff properties.
- Use linear and nonlinear least-squares fitting to squeeze the best results from your data.

2. SCHEDULE

1. *The First Week, 11 Feb to 17 Feb:* Finish §3 and understand §4. This involves considerable effort: taking 21-cm data on the sky and producing finished spectra involves not only data analysis and reduction (handout “Calibrating the Intensity and Shape of Spectral Lines”), but also converting among several spherical coordinate systems (alt-az, equatorial, Galactic) using rotation matrices (handout “Ay120a,b Cheat-Sheet for Spherical Coordinate Transformation”).

2. *The Second Week, 18 Feb to 24 Feb*: Finish §5. This involves measuring wavelengths in coaxial cable and waveguide. Reducing these data will, for the coax cable, provide your first experience with least-squares fitting (handout “Least-Squares Lite for the Budding Aficionado: Art and Practice”, sections 2-4); and, for the waveguide, your first experience with *nonlinear* least-squares fitting (again, the *Aficionado* handout, section 7).
3. *The Third Week, 25 Feb to 3 Mar*: Write up your formal report. And finish what you couldn’t get done during the first two weeks!

3. OUR FIRST RADIOASTRONOMICAL SPECTRA: THE 21-cm LINE WITH AN SSB MIXER (First Week)

This week we will use the techniques we explored in lab 1 to measure the 21-cm line (colloquially known among astronomers as the *HI*—that’s *H* - “*Roman numeral I*”, pronounced “*H-one*”) line shape, velocity, and intensity using the big horn on Campbell’s roof. Most of the measured power comes from our own electronics, not the HI line. It’s often called “noise”, and we need to get rid of this instrumental contribution.

We will use a double-heterodyne system for the 21-cm line at 1420.4 MHz. ‘Double heterodyne’ means we have two mixing stages. It works like this:

1. For the first mixer, we use the ordinary DSB technique. We use an l.o. (the ‘first local oscillator’) at 1230.0 MHz so that the difference frequency is 190.4 MHz and the sum frequency is 2650.4 MHz. We use a bandpass filter centered at 190 MHz to obliterate the sum frequency, so we are left with a replica of the 21-cm line that is centered at 190.4 MHz instead of 1420.4 MHz.
2. For the second mixer, we use the SSB technique. The second l.o. has frequency 190.0 MHz, so the output frequencies are positive and negative, centered about zero; this is called ‘baseband’. In the absence of Doppler shift, we are left with the line centered at 0.4 MHz. However, the line is shifted and broadened by Galactic rotation and the Earth’s orbital velocity, so it covers a range of frequency of, for example, 1420.2 ± 0.5 MHz. We use a low-pass filter with a cutoff frequency of, say, 2 MHz to eliminate aliasing, sample the complex signal, and Fourier transform to calculate the power spectrum. We take many power spectra and average them to reduce the noise.

3.1. The Measurements

Before beginning measurements in earnest, it's important to make sure the signal levels are OK and to experimentally determine which way frequency increases on your power spectra. So before getting out there in the rain to point the horn:

1. Set the system up as you will be using it to observe. Point the horn to zenith to reduce interference and thermal noise. Take some data. How fast must you sample?
2. Look at the range of sample values by plotting a bunch of them. Best is to make a histogram, which you can do using IDL's `histogram` function. We have a convenient 'wrapper' called `histo_wrap`, which you may find easier to use¹. The sampled numbers should cover plenty of bits; quantization should be only barely, or not at all, visibly evident in the histogram. Adjust `voltTreshold` in the call to `picosampler` accordingly.
3. Insert a test signal so that it appears in the upper sideband and take some data; then change the test signal frequency so it's in the lower sideband. You'll use this to determine whether the frequency axis is flipped or not.

Having determined that the system works, it's time to do astronomy! You could point the horn to a desired position, say a particular position in Galactic coordinates. Galactic coordinates are designated by the letters (l, b), which stand for Galactic longitude and latitude, respectively. Alternatively, you could use whatever position happens to be overhead and point the horn straight up; then you can use rotation-matrix techniques to figure out where you were pointed. We suggest this latter alternative.

We'll take two types of measurement: one to measure the line shape and the other to calibrate its intensity.

1. The power spectrum shape is dominated by the frequency-limiting filters. To see the line, which is weak, we need to correct for these filter shapes, which we do by having a spectrum containing no line. Accordingly, to get the shape we take two spectra: one with the line present (the 'online' spectrum) and one with the line not present (the 'offline' spectrum). We could obtain the online spectrum by centering the line and making a measurement; then changing the first l.o. frequency so that the line shifts

¹`histogram` is a native IDL function, so to get its documentation type `?histogram`. In contrast, `histo_wrap` is our own, in-house creation; for documentation of such non-native IDL functions, type `doc histo_wrap`. (There's another wrapper for plotting called `hist`, but we find it too complicated.)

either completely outside the band (this is called ‘frequency switching’), or partway over but is still in the band (called ‘in-band frequency switching’). The latter is nice because you are always looking at the line, thus getting better sig/noise. Take the two spectra with comparable integration times:

Take a spectrum with the line roughly in the upper half of the baseband spectrum, and another with it centered roughly in the lower half. Use the first as the ‘online’ and the second as the ‘offline’ spectrum for the upper half. Similarly, for the lower half, use the second as the ‘online’ and the first for the ‘offline’. (The HI line frequency is 1420.4058 MHz).

The line is weak, and you’ll need to take lots of spectra. Recall that `picosampler` obtains a specified number of spectra (`nSpectra`) each of which has 32000 datapoints and writes them all out into a single file. Suppose you decide to obtain 200000 spectra. You might be tempted to do it in one fell swoop by setting `nSpectra=200000`. That would not be a good idea because the file size would exceed your machine’s memory capacity. You’d need to keep `nSpectra` ≤ 4096 and call `picosampler` the requisite number of times (in this case, about 50).

2. Intensity calibration requires a second pair of measurements, which can be short. Easiest is probably to take one with the horn looking at a known blackbody and one looking at cold sky. What’s a convenient blackbody? You! And your friends! So take one short measurement with the horn pointing straight up at cold sky and the other with as many people as you can find standing in front of it and ‘filling the aperture’. Call these spectra $s_{coldsky}$ and s_{300K} .

3.2. The Analysis

For the reduction:

1. Make a plot or a histogram of some representative voltage values in one of your three datasets to make sure that they are, indeed, not too quantized and don’t exceed the maximum range of the ADC. The histogram shape should look like a well-known function. Which function? Does it?
2. Consider, first, the s_{online} and $s_{offline}$ spectra, from which you can find the line shape. There are many such spectra, 200000 in our above example. You need to combine these to make a single spectrum for each measurement. You can do this by averaging (use IDL’s `total` function) or by taking the medians (use IDL’s `median` function).

The former gives a less noisy result, but the latter handles time-variable interference better; use both and compare the results. Even after combining the 200000 spectra, the resultant spectrum will still look noisy. You can reduce the noise by averaging over channels—by ‘smoothing’ the spectrum. This reduces the noise, but degrades the spectral resolution, so you have to make a compromise on how many channels to smooth over. To decide, realize that the HI line is never narrower than about 1 km/s, so it’s OK to degrade the frequency resolution to, say, 1 or 2 kHz. Again, you do the smoothing by averaging or median, and again try both to see what happens.

In the smoothed s_{online} spectrum you won’t be able to see the HI line, because the instrumental bandpass dominates the spectrum shape. The instrumental bandpass is determined mainly by the low-pass filter, which should fall smoothly to zero as the frequency increases. Does it? If not, should you worry about aliasing?

3. You can remove the instrumental bandpass, and thus get the shape of the line (but not the intensity), by taking the ratio

$$ratio = \frac{s_{online}}{s_{offline}} . \quad (1)$$

This is the *first* factor (i.e., the shape) in equation (13) in the spectral-line handout.

4. To get the line intensity in terms of the calibration noise source, multiply the ratio spectrum by T_{sys} —the second factor in equation (13). We obtain T_{sys} by combining the 300K and caloff spectra, as per equation (15) in the handout:

$$T_{sys} = \frac{\sum s_{coldsky}}{\sum (s_{300K} - s_{coldsky})} T_{cal} \quad (2)$$

Here, a sum is over all channels in a spectrum. $T_{cal} = 300$ K, because that’s the thermal power you injected by standing in front of the horn. Then the final, intensity-calibrated spectrum is equation (16) in the handout, namely

$$final\ calibrated\ spectrum = ratio \times T_{sys} \quad (3)$$

5. Plot your final spectrum versus the r.f. frequency.
6. Plot your final spectrum versus the Doppler velocity². Remember that, by astronomical convention, positive velocity means motion away (remember the expansion of the Universe!), so

²Astronomers usually express velocities in km s⁻¹.

$$\frac{v}{c} = -\frac{\Delta f}{f_0}, \quad (4)$$

where c is the speed of light and Δf is the frequency offset from the line frequency f_0 ³.

7. You may think we’ve done it all by this point, but we haven’t! We need to correct the observed velocity for the orbital velocity of the Earth, and also the Earth’s spin. And when observing the Galaxy, it is customary to express velocities with respect to the ‘Local Standard of Rest’ (LSR), so that’s yet another correction.

Calculate the Doppler correction using `ugdoppler.pro` (for documentation use `doc` or `doc_library`). Correct the velocities and plot the spectra for two reference frames: (1), the Sun or barycentric reference frame, which gives velocity with respect to the Sun; and (2) the Local Standard of Rest (LSR), which is approximately the frame that would rotate around the Galaxy in a circular orbit.

Notes on ugdoppler:

- (a) You need the celestial coordinates of the source, (ra, dec) . How to find these for the horn pointing straight up? Use rotation matrices! See the handout “Ay120a,b Cheat-Sheet for Spherical Coordinate Transformation”.
- (b) You need the Julian day of the observation; see §4.
- (c) You need the observatory coordinates (north latitude and west longitude) in degrees; you could enter them with the pair of optional input parameters (`nlat`, `wlong`), but you don’t have to because the default values are Campbell Hall’s values.

3.3. For Your Lab Report

Create relevant plots and commentary. In addition, make a block diagram of the receiver, including all mixers, oscillators, and filters.

³Radio astronomers, being frequency-oriented, use this equation; optical astronomers, of course, use something different: $\frac{v}{c} = \frac{\Delta\lambda}{\lambda_0}$. At high redshifts the difference becomes significant; the optical definition is the usual standard.

4. CIVIL TIME, UNIVERSAL TIME, SIDEREAL TIME, JULIAN DAY, MODIFIED JULIAN DAY

4.1. Definitions

Coordinated Universal Time (UTC) is the Civil Time⁴ in Greenwich, which is 8 hours ahead of (larger than) PST. Think of Civil time as Solar time, because the 24-hour period is the same as that of the Sun.

Think of Local Sidereal Time as “star time”; it tells when a particular star (not the Sun) is visible. It’s period is a bit less than 24 hours—4 minutes less, almost exactly.

The Julian Day begins at noon (UT=12hr) in Greenwich. JD=0 occurs on 1 Jan –4713, 12 hr UT. The JD begins at midnight on the international date line, so it is 12 hours out of phase with UT. The Julian day is a double-precision float: it contains the time as the fractional part of the day. It’s not just an integer day number. The current JD is about 2457062.

The Modified Julian Day is a shorter version of Julian Day. MJD=0 occurs at 0 hr UT on 17 Nov 1858. The MJD begins at midnight in Greenwich and at noon on the international date line, so MJD lines up properly with UT. The current MJD is about 57062.

4.2. Some Useful IDL Procedures

These routines are based on GSFC `ct21st.pro`. LST tells when a star is overhead, so its rate of passage depends on the Earth’s spin. The Earth is constantly slowing owing to tidal friction produced by the Moon. The conversion between Civil Time and LST has to be adjusted periodically by inserting ‘leap seconds’, so unless you keep up with this, the conversion will be off by a few seconds.

You need to know your longitude to calculate LST. NCH has `nlat=37.8732`, `elong=-122.2573` (same as `wlong=+122.2573`). For all procedures mentioned here, these are the defaults.

```
local_now = SYSTIME() gives current local time as a string
```

```
ut_now = SYSTIME(/ut) gives current UTC as a string
```

⁴You use Civil Time when you set your alarm clock for getting up in the morning.

`ut_sec_since = SYSTIME(/sec)` gives seconds elapsed since 1 January 1970 UTC

`julian_now = SYSTIME(/julian, /utc)` gives current julian day (which contains the current time, too--it's not just an integer number. the `/utc` is required for it to give julian date relative to Greenwich, which is the only one that means anything.

`LST --` prints LST right now on the screen

`lstnow = LSTNOW()` -- returns the current LST

`lst_julian = ILST(julian=juldate)` -- returns the LST for the specified Julian day

`CALDAT, Julian, Month [, Day [, Year [, Hour [, Minute [, Second]]]]]`
Given the Julian day (which is defined relative to Greenwich), this returns the civil time at Greenwich.

`julian = JULDAY(Month, Day, Year, Hour, Minute, Second)`
Given the civil time at Greenwich, this returns the Julian day.

`modified_julian_day = MJD2JD(julian_day, /reverse)`

`julian_day = MJD2JD(modified_julian_day)`

5. VSWR MEASUREMENTS WITH SLOTTED LINES AND WAVEGUIDES (Second Week)

How do you move power or electronic information from one place to another? Transmission lines (cables) and waveguides are indispensable. Sounds easy—just connect two things with a wire. But does this really work? We'll explore cables, waveguides, and reflections by measuring the Voltage Standing Wave Ratio (VSWR), which results from interference of incident and reflected waves.

The experimental work and measurements described in §5 should be done by groups.

The analysis in §6 should be done by individuals. The analysis is nontrivial, so don't delay with the measurements!

And with all this, you need to pursue a bit of theory. It is essential to look at the discussion in §7, which discusses VSWR and also tells how a waveguide works; you can't interpret your experimental results without these. §8 goes further, referencing sections of the wonderful book by Ramo, Whinnery, and van Duzer. We briefly summarize those sections with short descriptive discussions, and you should at least read these summaries.

5.1. The coax slotted line at C-band (~ 3 GHz)

Set up the slotted coaxial line with the wide-range HP 83712B synthesizer. With the far end of the slotted line open, find the wavelength in the cable by measuring the positions of *all* nulls as accurately as you can. Do the same with the far end shorted and note the differences. In particular, note how the positions of the nulls change when the slotted line is terminated with an open versus when it is shorted. Why is this?

From your measurements calculate the velocity [what does “velocity” mean?] in the slotted line by comparing the wavelength and the frequency. First do a quick calculation using your measurement's. Then do it as accurately as possible by using least-squares fitting and following the advice in §6.1. *Note:* As it happens, the velocity is independent of frequency for a coax cable—in contrast to a waveguide. If you are so motivated, you can check this experimentally.

Put a matched load, and then a calibrated mismatched load, on the end of the slotted line and measure the VSWR using a few null/maximum pairs. Also use an ohmmeter to measure the DC resistances of the two loads, and determine whether your results agree with the theory in §7.1; while you're at it, measure the DC resistance of the cable and ponder why it is called “50-ohm cable”.

5.2. The X-band waveguide (7 to 12 GHz)

Now we'll do the same as we did in §5.1, but for X-band waveguide. The cutoff frequency for X-band waveguide is about 7 GHz. Pick a suitable frequency and, with the far end of the waveguide completely open, measure the VSWR; then short the end of the waveguide, maybe with a piece of aluminum foil or a metal plate, and repeat. Why is an open ended waveguide different from an open ended coax cable?

With the end shorted, measure the velocity in the slotted waveguide by comparing the wavelength and the frequency; do this for several (half-dozen?) frequencies, including

one or two near cutoff, and compare with theory. These frequencies should be reasonably closely separated, say by no more than 1 GHz. From these measurements, derive the cutoff wavelength (and thus the waveguide width) as accurately as you can. Also measure the waveguide dimensions, and compare with the width you derived above. Finally, play around with the cutoff frequency behavior and, if you have time, see how the transmission behaves in the vicinity of cutoff.

5.3. EXTRA—for those Without A Life...

Attach a reasonably long piece of coax cable to the far end of the slotted line and, using the slotted line, measure the wavelength (and therefore velocity) *in the flexible coax*. It is easy to measure the wavelength in the *slotted* coax, but it requires real insight and cleverness to determine the wavelength in the *flexible* coax. For this, a long cable, a few meters or more, is helpful.

Hint: To make this measurement, you need to change a parameter and observe the results of the change. What parameters can you change? Which one allows you to do the job? (*HEY!!!* You're *NOT ALLOWED* to change the length of the cable by cutting it!!!!)

6. GETTING THE MOST OUT OF YOUR DATA: STATISTICAL ERROR ANALYSIS (Second Week)

6.1. The coax slotted-line wavelength

For the measurements of §5.1, you have sampled M cycles of the standing wave. You can calculate the wavelength from the distances between a single null pair. While this calculation is a good estimate, each of your measurements has an error. You have measured the distance between $(M - 1)$ null pairs. What's the best way to combine these measurements so as to obtain the most accurate wavelength?

(1) One way is to calculate the wavelength from each neighboring null *pair* and take the average of the $(M - 1)$ *pairs*. You might then apply the usual rule to obtain the uncertainty in the average, i.e. the uncertainty is $\frac{(\text{standard deviation})}{(M-1)}$. However, this would *not be correct*. Why? (Hint: The answer you get by averaging the $(M - 1)$ pairs is identical to that obtained from a *single* well-chosen non-neighbor pair. Which one?). Actually, this method gives *almost* the best estimate of the wavelength. But you can do better!

(2) The positions of the nulls should increase linearly with distance. So if x_m is the position of null number m , we should have

$$x_m = A + m\frac{\lambda}{2}, \quad (5)$$

You know x_m and m from measurement and you would like to derive A and λ . This is a classic least squares problem. Use the handout “Least-Squares Lite for the Budding Aficionado: Art and Practice” to write your own least-squares fit program using matrix techniques (see §2 of that handout).

6.2. The X-band waveguide

Here we want to do a least-squares fit of equation 13 to solve for the waveguide width a . Now, you’ve already measured a with the caliper. But here the idea is to compare your measurements of the waveguide cutoff with what theory (that’s equation 13) predicts. So you imagine that a is unknown and determine it from your data on λ_g versus λ . Doing this is not as straightforward as above in section 6.1 because λ_g depends *nonlinearly* on a . You can handle this problem in two ways: either the “brute force” method of section 9, or the more general nonlinear treatment in §7 of the “Least-Squares Lite” handout). The “brute force” method is simpler, and I recommend using it here—we’ll get opportunities later to be more elegant.

From your above solution, determine the best-fit value of a and compare it with the value you measured with the caliper.

7. THEORETICAL COMMENTARY

The material in this section is essential for your work in this lab. It is extracted from the excellent book by Ramo, Whinnery, and van Duzer (RWvD). If you are so inclined, take a look at that book; we provide commentary on the book in §8.

7.1. The Voltage Standing Wave Ratio (VSWR)

The voltage reflection coefficient for a load with resistance R_L that terminates a line with impedance Z_0 is

$$\rho = \frac{V_R}{V_F} = \frac{R_L - Z_0}{R_L + Z_0} \quad (6)$$

where V_R is the reflected voltage amplitude and V_F the forward, or incident, one. The reflected voltage interferes constructively with the forward one, giving a maximum $V_{max} = V_{forward} + V_{reflected}$; and destructively, giving a minimum $V_{min} = V_{forward} - V_{reflected}$.

In more detail, we can write

$$V_{forward} = V_F e^{i2\pi(ft-x/\lambda)}, \quad (7)$$

which travels towards positive x , and

$$V_{reflected} = V_R e^{i2\pi(ft+x/\lambda)}, \quad (8)$$

which goes the other direction. These add to give

$$V_x = V_F e^{i2\pi ft} [e^{-i2\pi x/\lambda} + \rho e^{i2\pi x/\lambda}] \quad (9)$$

which can be written as

$$V_x = V_F e^{i2\pi ft} \left\{ \frac{1+\rho}{2} [e^{i2\pi x/\lambda} + e^{-i2\pi x/\lambda}] - \frac{1-\rho}{2} [e^{i2\pi x/\lambda} - e^{-i2\pi x/\lambda}] \right\} \quad (10)$$

or, in the conventional trig notation that we all understand,

$$V_x = V_F e^{i2\pi ft} \{(1+\rho) \cos(2\pi x/\lambda) - (1-\rho) \sin(2\pi x/\lambda)\} \quad (11)$$

So the standing wave pattern repeats every wavelength with an amplitude and phase (position) that depend on ρ . With our slotted-line probe, we measure the mean *absolute value* of the internal electric field, which is proportional to $|V_x|$. As we move along in x , $|V_x|$ oscillates sinusoidally between values $|V_{max}|$ and $|V_{min}|$ with period λ . The VSWR is the max-to-min ratio (see RWvD §8.8, equation 11)

$$VSWR = \frac{|V_{max}|}{|V_{min}|} = \frac{1+\rho}{1-\rho} \quad (12)$$

7.2. The TE_{10} Mode in a Rectangular Waveguide.

RWvD provide a beautiful discussion of the TE_{10} mode in common rectangular waveguide (in which the ratio of side lengths equals 2). Most important is their equation 11, which gives the “guide wavelength” λ_g ; we reproduce it here:

$$\lambda_g = \frac{v_p}{f} = \frac{\lambda}{[1 - (\lambda/2a)^2]^{1/2}}, \quad (13)$$

where λ is the free-space wavelength. The guide wavelength is the wavelength within the waveguide, and combining it with the frequency gives the propagation velocity in the guide.

8. REFERENCE READING

You can do this lab without doing any reading... but then you wouldn't gain the insight from the highly readable and well-written text by Ramo, Whinnery, and van Duzer (RWvD), *Fields and Waves in Communication Electronics*. This is a great book because it contains commentary on applications to modern devices (e.g., the discussions of transmission lines carrying pulse from one piece of computer hardware to another are very nice).

The notes below refer to the second edition. If you have the time and inclination to read, we suggest that you concentrate on the starred items below (***) . The italicized numbers refer to the very same sections of the book.

8.1. Theory of Coaxial Cable

*** *RWvD §5.2: Introduction: what transmission lines do.* Basic equations for transmission lines. Note equation (14) for the characteristic impedance Z_0 , which is the ratio of voltage to current in the line. This is expressed in ohms, but it is not “ordinary resistance” because voltage and current are out of phase. Example 5.2 calculates Z_0 for a coax line. RG58/U, done in the example, has 50 ohms; so do the cables we use in our system (in contrast, the TV standard is 75 ohms). Because of the dielectric in the cable, the wave (phase) velocity is ~ 0.7 that in free space, making the wavelength in the cable shorter than that in free space; one needs to account for this when using cables to produce delays in a signal, as in an interferometer or a quadrature circuit.

*** *RWvD §5.4: Reflection and transmission at a resistive discontinuity.* The purpose

of a transmission line is to take power from a source to a load. The load should absorb the power and therefore should be resistive. If its resistance matches the line impedance, then all the power transmitted by the line is absorbed by the load. If not, some is reflected; see equations (4) and (6). We need to tune our devices so that they are “matched” and reflect no power.

RWvD §5.6: Standing Wave Ratio. Finally: how do we measure mismatch? RWvD’s example 5.6 discusses the measurement of slotted line impedance measurements, which we will do in the lab. (This only appears in the Second Edition photocopy of RWvD.) With the slotted line, the probe samples the electric field within the cable, which is proportional to the voltage. Thus we measure the ratio $\frac{V_{max}}{V_{min}}$ in equation 12, from which we calculate *VSWR*—which is related to the power reflected in RWvD’s equation (6).

8.2. The Theory of Waveguide

*** *RWvD §8.1: Introduction: general description of waveguides.*

*** *RWvD §8.8: The TE_{10} Mode in a Rectangular Guide.* A nice, complete discussion of the TE_{10} mode in common rectangular waveguide (in which the ratio of side lengths equals 2). Most important is equation (11) (same as our equation 13 above), which gives the “guide wavelength” λ_g .

In this mode, E is perpendicular to the long side of the guide (and also to the longitudinal axis of the guide). We speak of the polarization of the waveguide: the guide transmits linear polarization that is perpendicular to the long side of a waveguide. When we couple power into a guide from coax, we do so with a probe that is parallel to the internal electric field. When we use a sliding probe to measure *VSWR* in a waveguide, the probe extends through a long slot in the broad side of the waveguide and the probe is parallel to the E field inside the guide, so it samples the internal E field. Note **Figure 8.8**, which shows the current flow in the waveguide walls; note that the current flow down the center of the broad side of the guide is parallel to the axis of the guide—that is, parallel to the slot that we use for the probe. The slot, being parallel to the current flow, does not interrupt any current flow and has no effect on the internal distribution of fields in the guide. Clever!

RWvD §8.10. Makes the analogy between coax cable and waveguide. A coaxial line can be considered as a waveguide containing walls not only the outside but also on the inside. The usual mode that propagates in coax is the TEM mode. A TE_{10} -like waveguide mode enters for wavelengths smaller than that given by equation (4); at this point, coax lines become basically unusable. For type N connectors, we are beginning to approach this limit

in our 12 GHz system!

RWvD §8.11. Brief description of how coax lines are coupled to waveguide. The method depends on which mode you wish to excite. For the TE_{10} mode, method (b) in Figure 8.11 works well and is almost universally the one used.

8.3. Theory of the Directional Coupler

Section 11.11 at the beginning (page 551, 552), and Figure 11.11, make it clear how these devices work. The clever use of phase to make a signal interfere constructively and/or destructively with itself is basic to many devices, some of which we will use.