Spectrograph Basics

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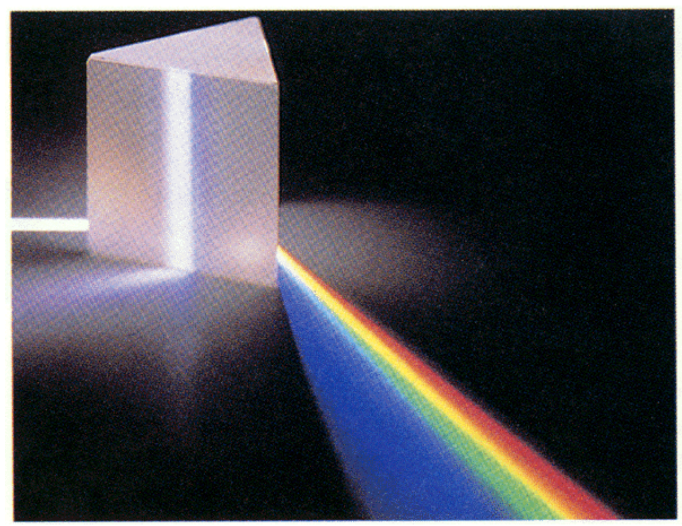
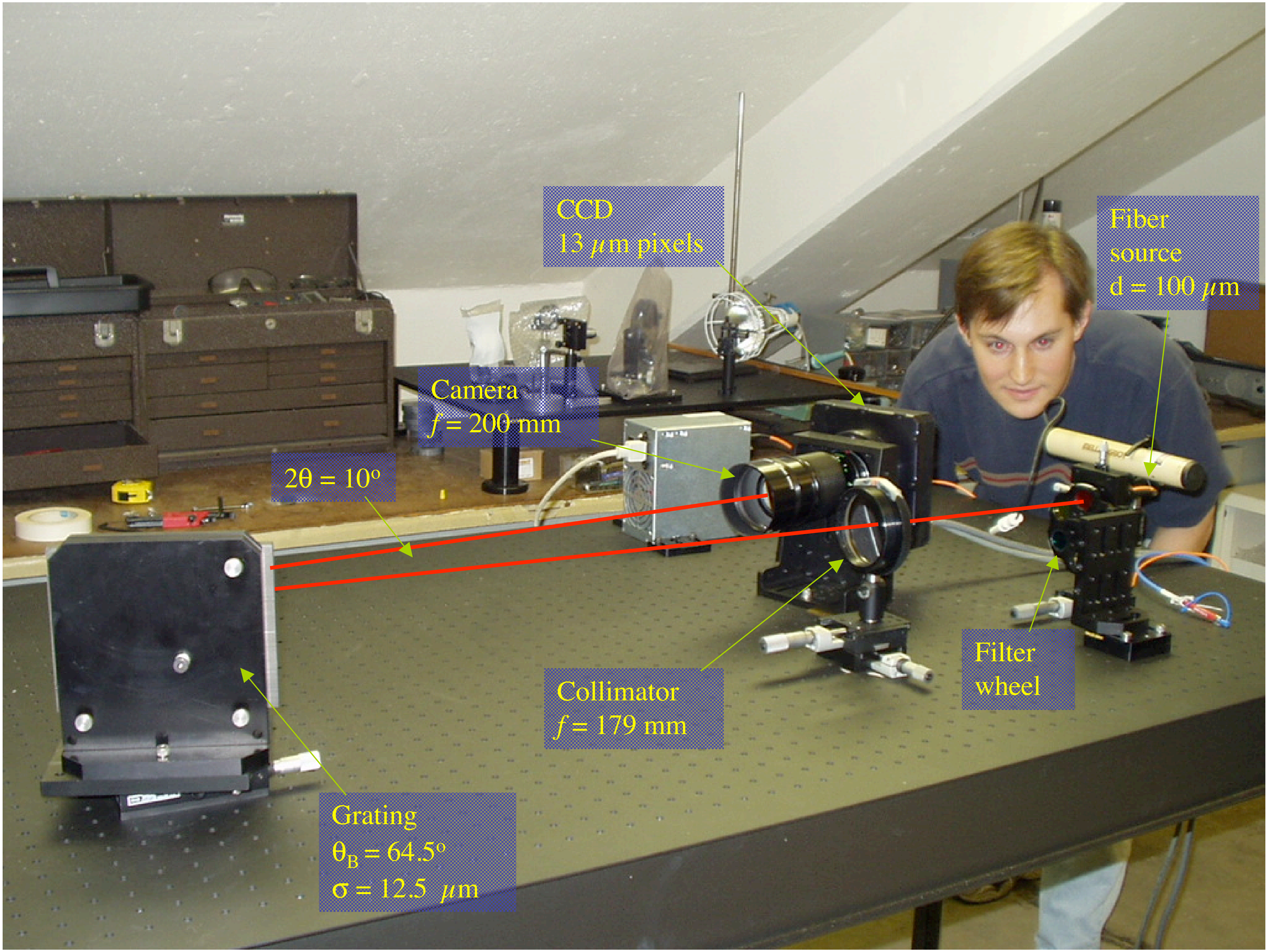
Dispersive Spectrometers

- Dispersive spectrometers are a class of instruments that encode wavelength as position on a focal plane detector
- Dispersion can be caused by refraction or diffraction
- Key element is
  - Prism ($dn/d\lambda \neq 0$)
  - Grating
- Gratings are favored
  - Flexible
    - Transmission or reflection
    - Groove spacing
    - Plane or powered surface
  - Efficient
    - Grating can be blazed
  - Lightweight

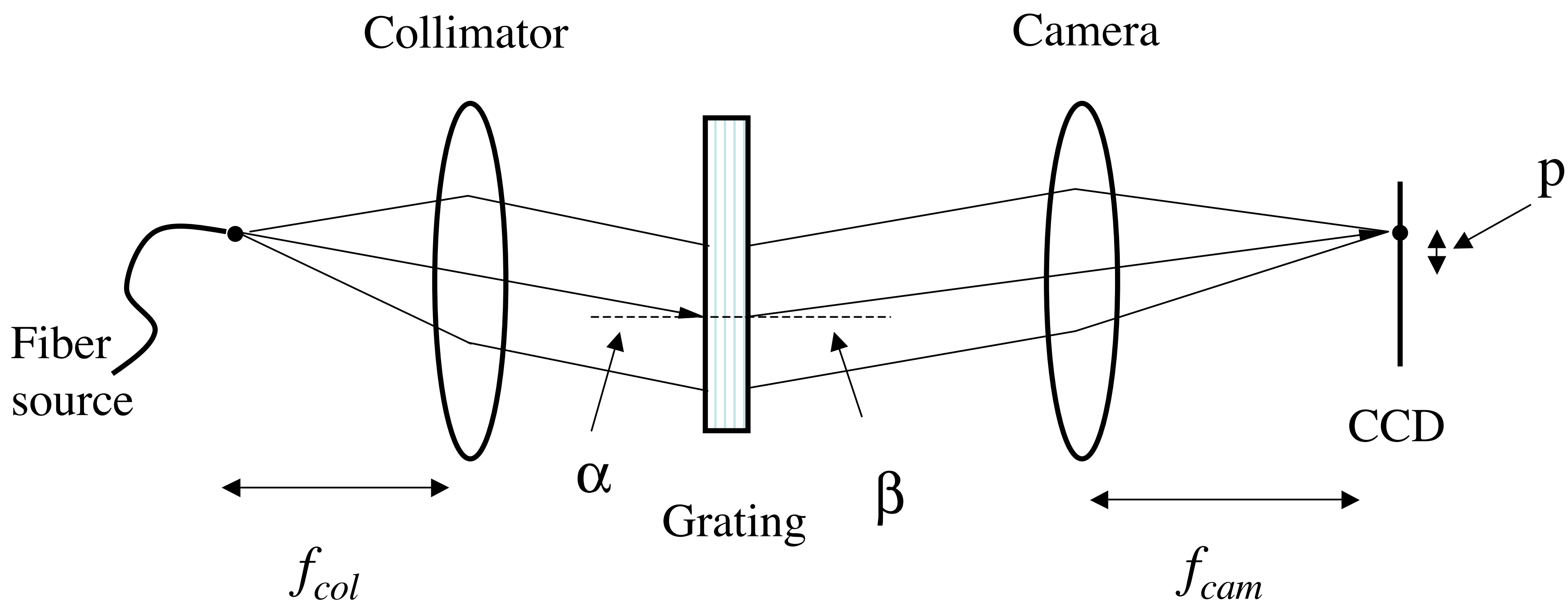


Spectrometers as Imagers

- A spectrometer is fundamentally a device which makes an image of a source
  - The position of the image of the source depends on wavelength
- Typically the spectrometer makes an image of an aperture or slit
  - In the solar spectrometer, the spectrometer makes an image of the light exiting an optical fiber
- The location and size of the image is determined jointly by the laws of geometric optics and the grating equation


CCD
13 μm pixels
Fiber
source
d = 100 μm
Camera
f = 200 mm
2θ = 10°
Collimator
f = 179 mm
Filter
wheel
Grating
θ_B = 64.5°
σ = 12.5 μm

Schematic Spectrometer



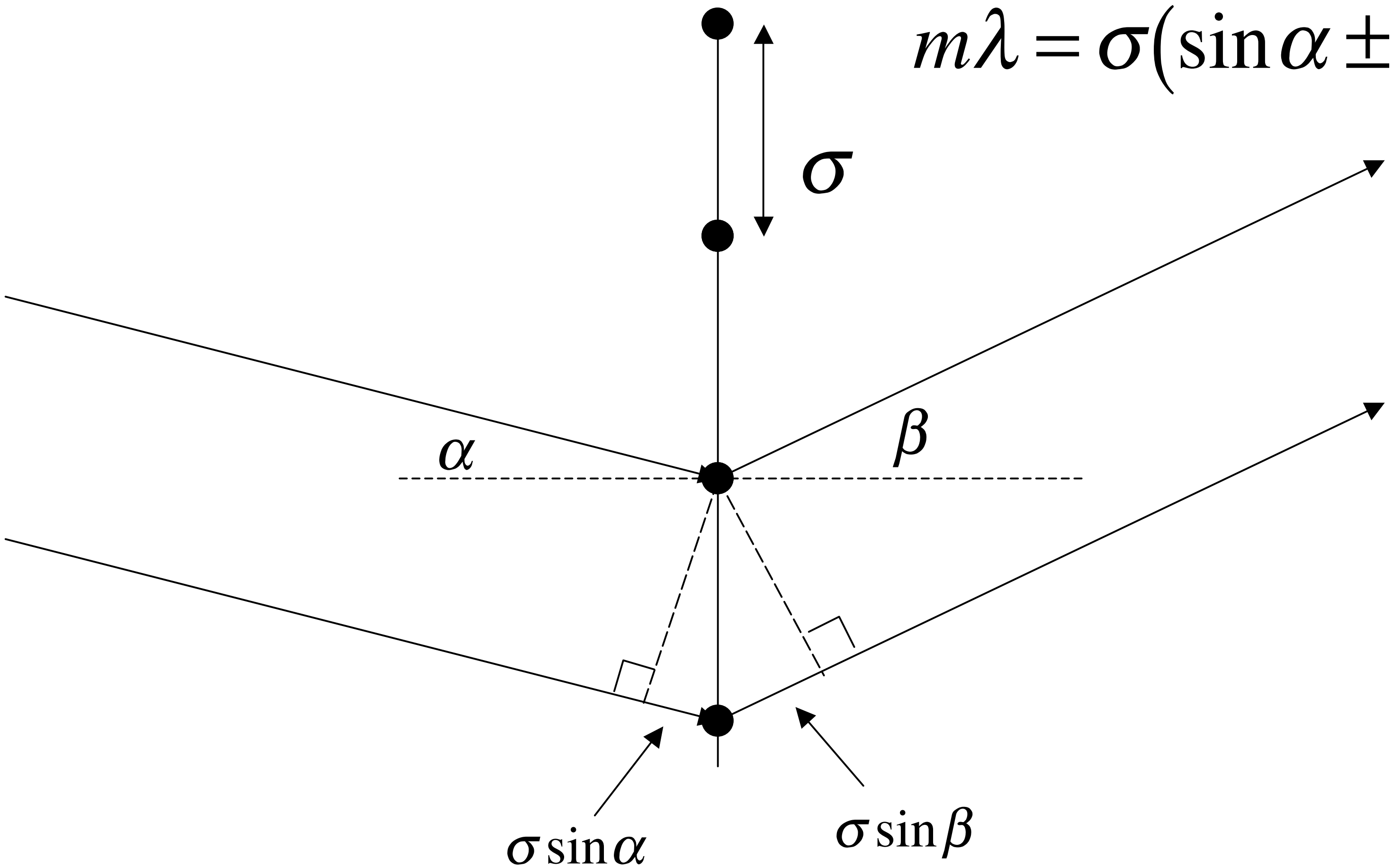
For a reflection grating α and β have the same sign of they are on the same side of the grating normal. For a transmission grating they have the same sign if the diffracted ray crosses the normal.

$$m\lambda = \sigma(\sin\alpha - \sin\beta)$$

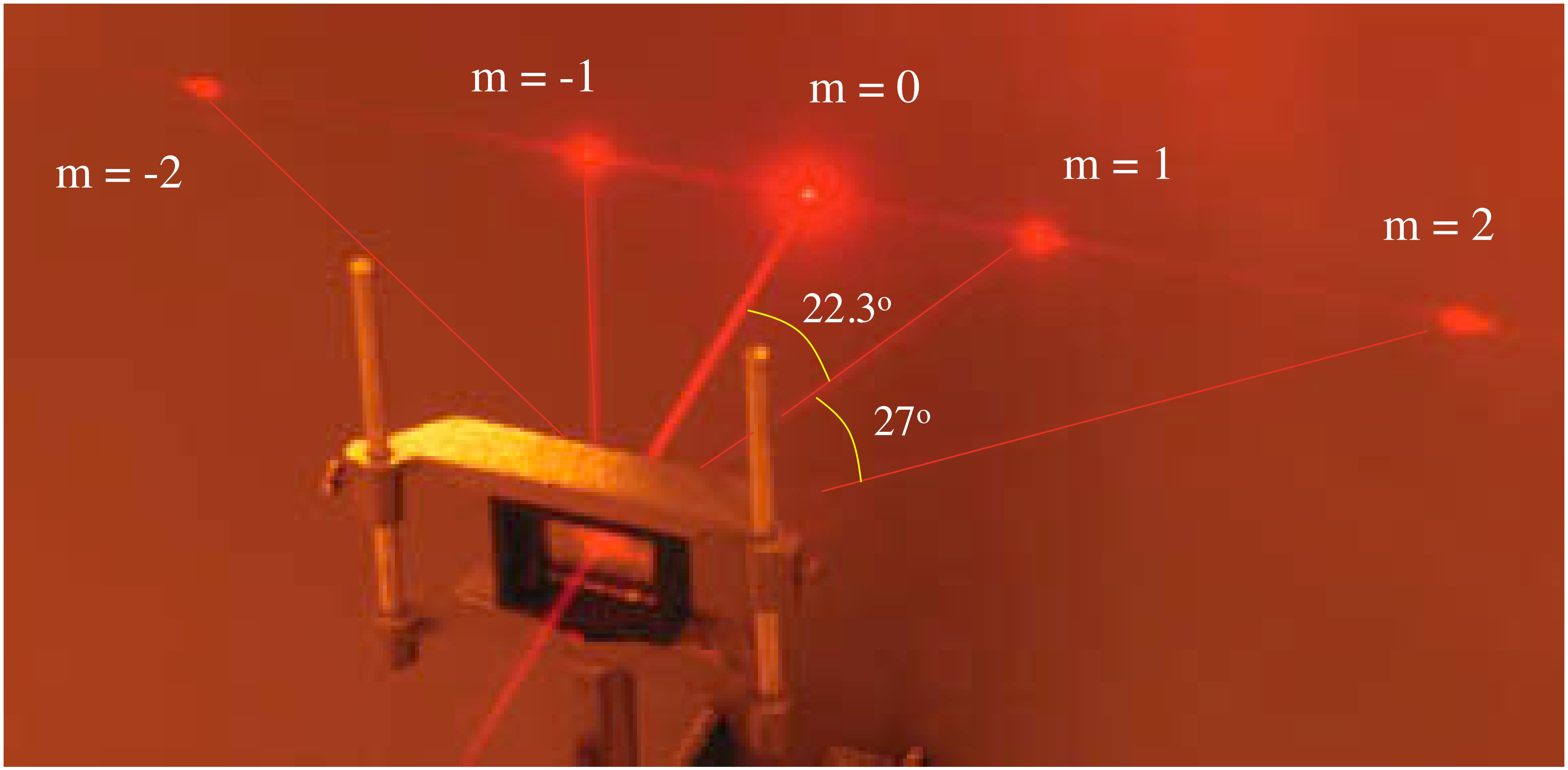
- for transmission
+ for reflection

Condition for Constructive Interference

$$m\lambda = \sigma(\sin\alpha \pm \sin\beta)$$

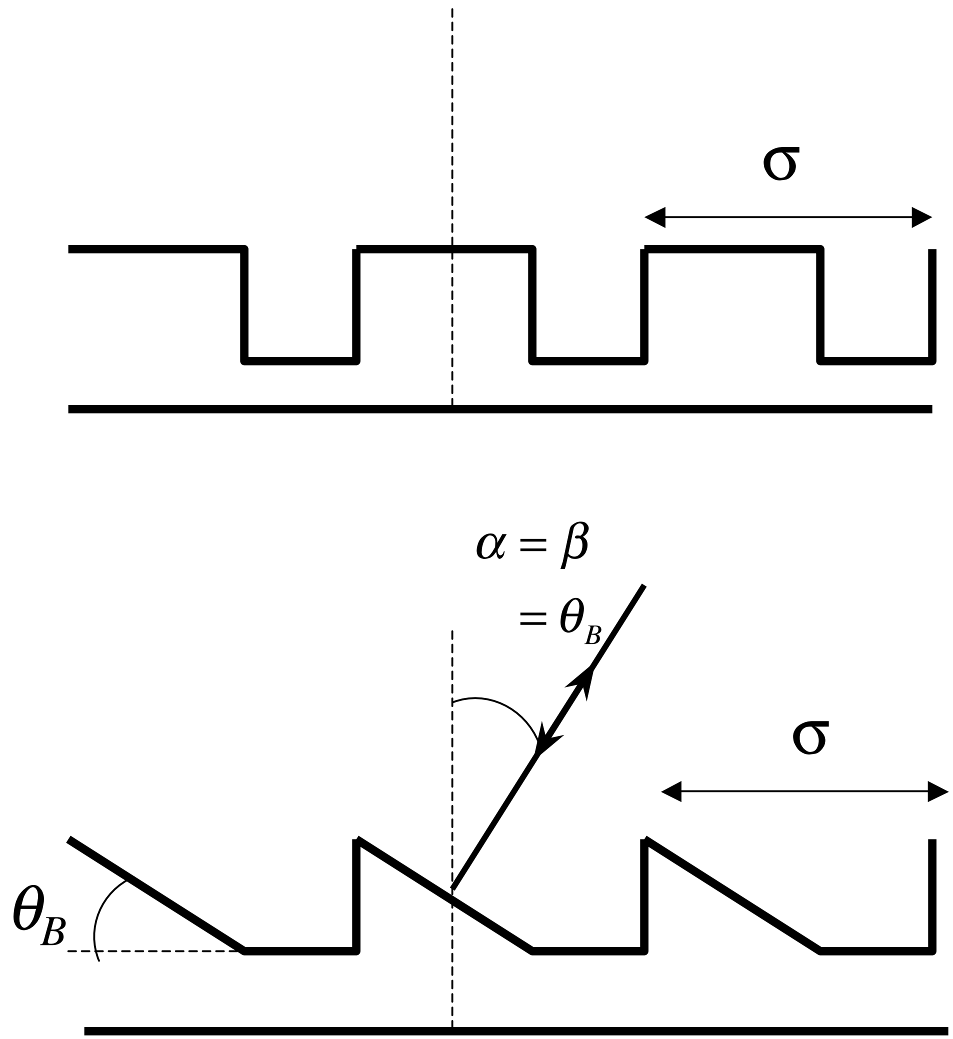


Orders



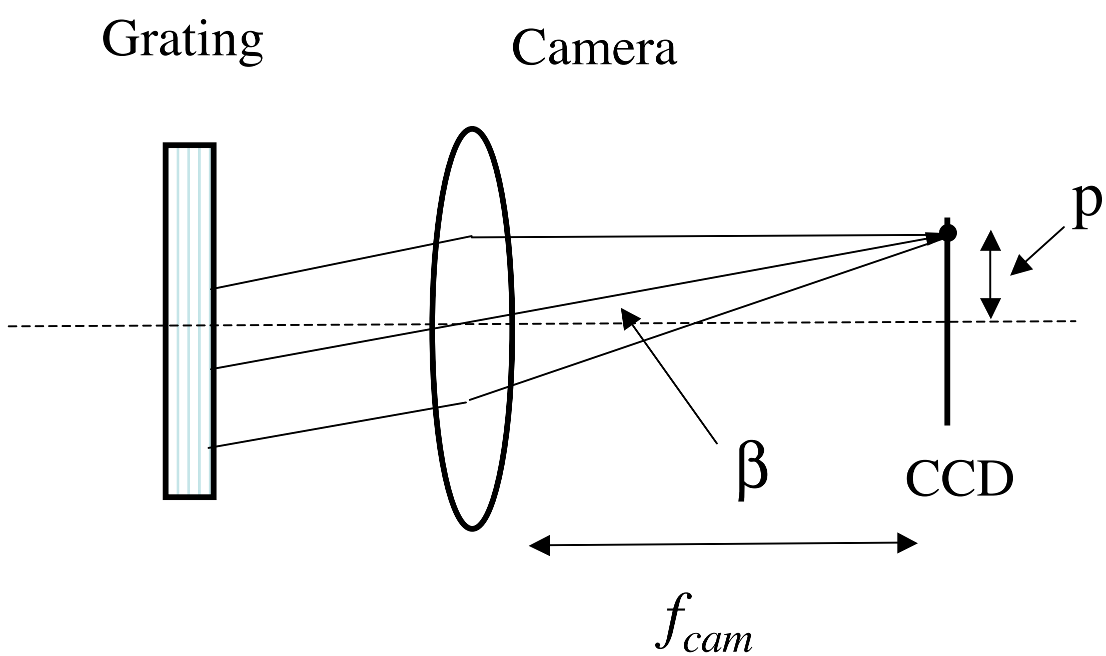
Transmission grating $1/\sigma = 600\ \text{mm}^{-1}$

Blazed Grating



Positional Encoding

- The angle is given by the grating equation and the position set by the camera focal length



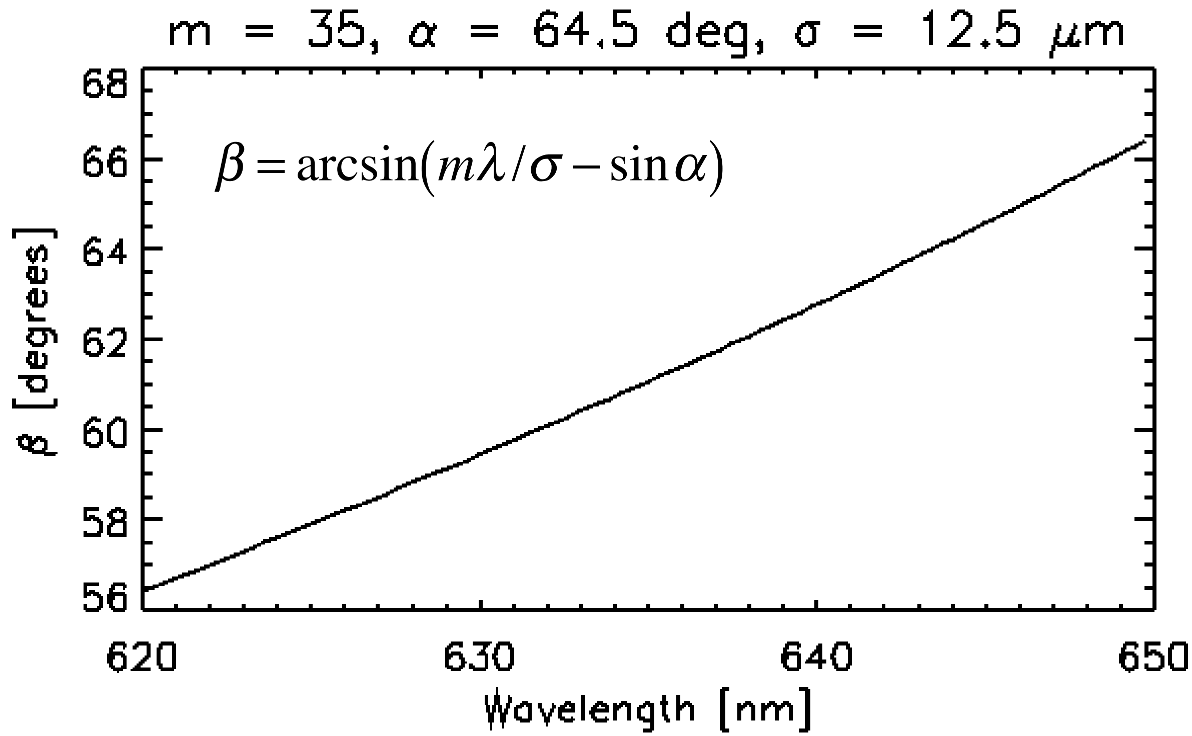
$$m\lambda = \sigma(\sin\alpha + \sin\beta)$$

$$\beta = \arcsin(m\lambda/\sigma - \sin\alpha)$$

$$p = f_{cam}\tan(\beta)$$

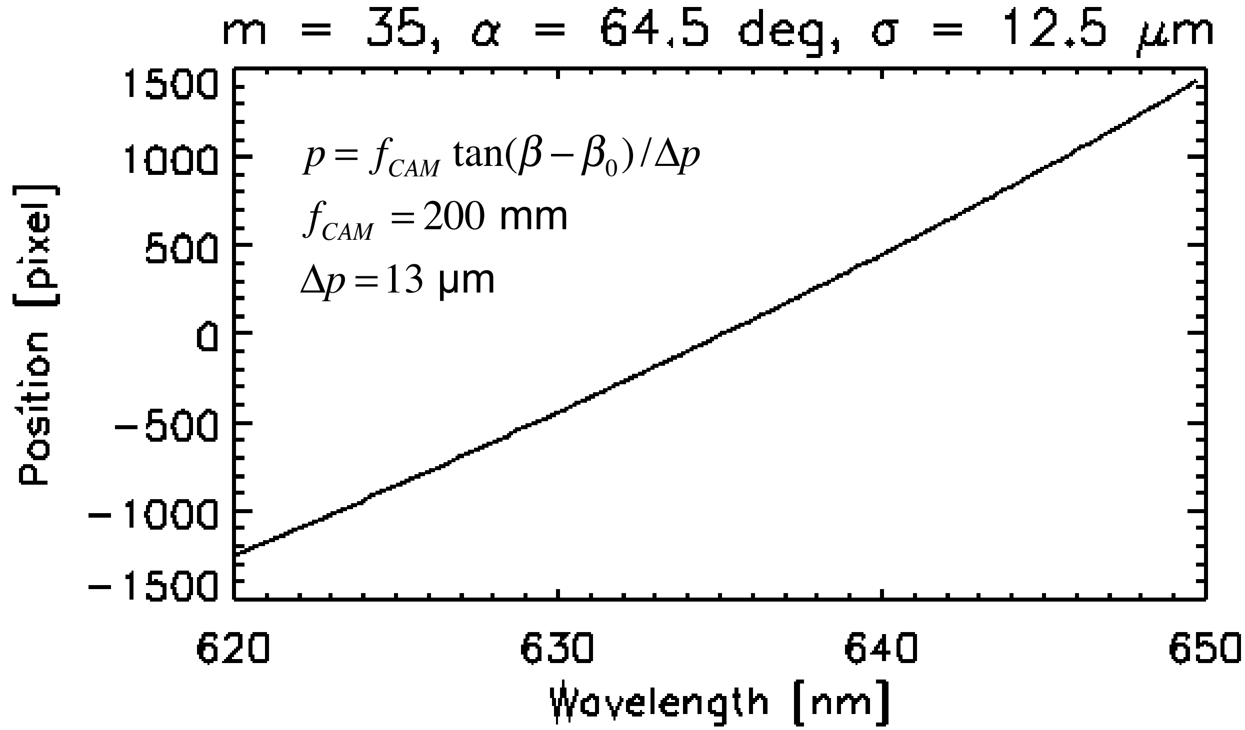
Mapping Wavelength to Angle

- Holding α and *m* constant, β varies with λ



Mapping Wavelength to Position

- Holding α and m constant, p varies with λ



Dispersion

- Dispersion gives the angular spread of diffraction, $\delta\beta$, for a source with wavelength spread, $\delta\lambda$
  - Start with the grating equation and hold the angle of incidence, α, and the order, m, constant

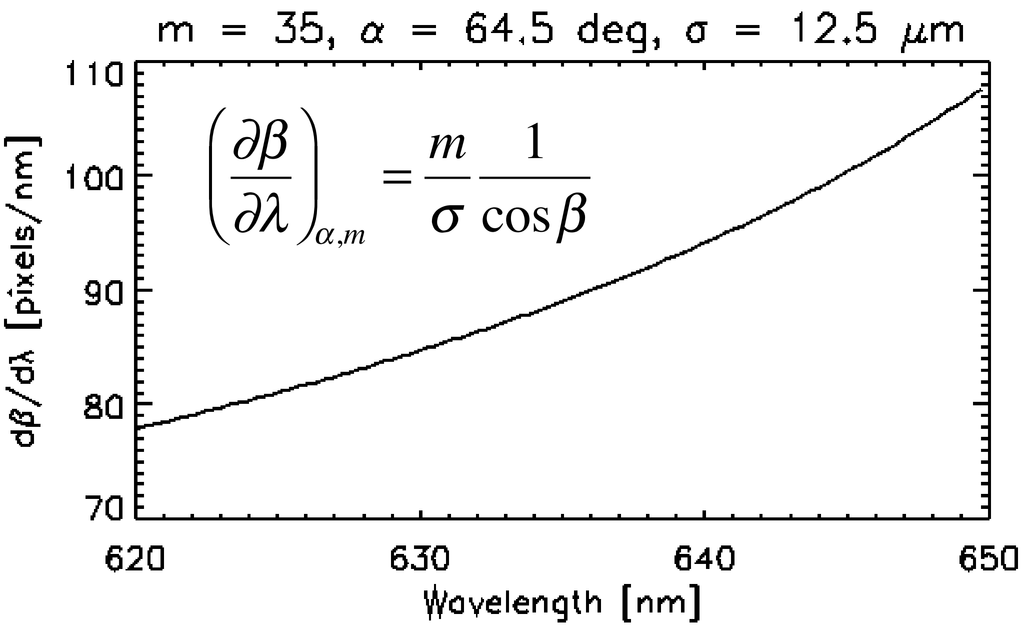
$$m\lambda = \sigma(\sin\alpha + \sin\beta)$$

$$m\,\delta\lambda = \sigma\cos\beta\,\delta\beta$$

$$\left(\frac{\partial\beta}{\partial\lambda}\right)_{\alpha,m} = \frac{m}{\sigma}\frac{1}{\cos\beta}$$

Dispersion

- Over a limited range of wavelength dispersion is ≈ constant
  - Linear relation between wavelength & position

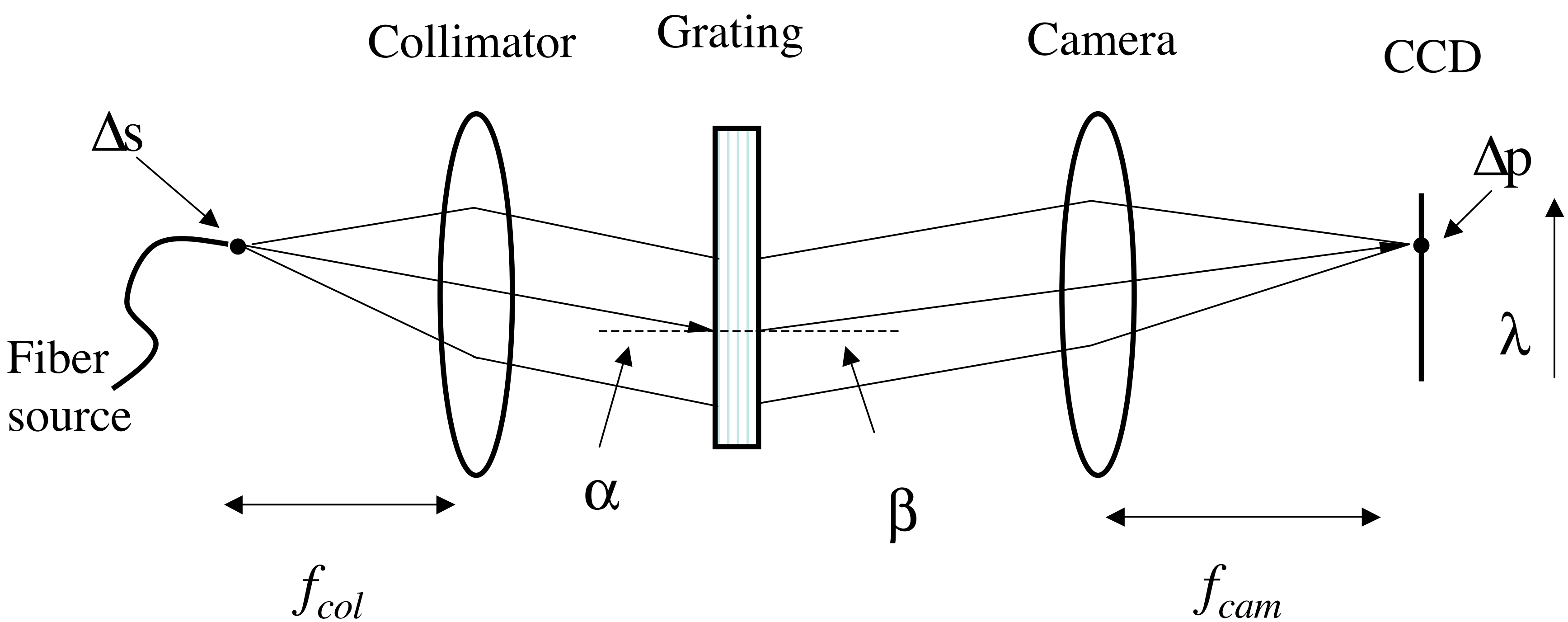


Dispersion

- With higher dispersion it is possible to distinguish closely spaced wavelengths
- High dispersion corresponds to
  - High order (large *m*)
  - Narrow grooves/high groove density
  - Large β ($\approx \pi/2$)

$$\left(\frac{\partial \beta}{\partial \lambda}\right)_{\alpha,m} = \frac{m}{\sigma}\frac{1}{\cos\beta}$$

Spectral Resolution



$$\Delta\alpha = \Delta s / f_{col}, \quad \Delta\beta = \left(\frac{\partial\beta}{\partial\alpha}\right)_{\lambda} \Delta\alpha, \quad \Delta p = f_{cam}\Delta\beta$$

Spectral Resolution: the Diffraction Limit

- Even if the input is a point source, the image has a finite size on the CCD array, Δp, due to diffraction
  - The angular size of camera images, $\delta\beta=\lambda/D_{cam}$, limits the spectral resolution

$$\delta\lambda = \frac{\partial\lambda}{\partial\beta}\delta\beta$$

$$= \frac{\sigma\cos\beta}{m}\delta\beta$$

$$\delta\beta = \frac{\lambda}{D_{cam}}$$

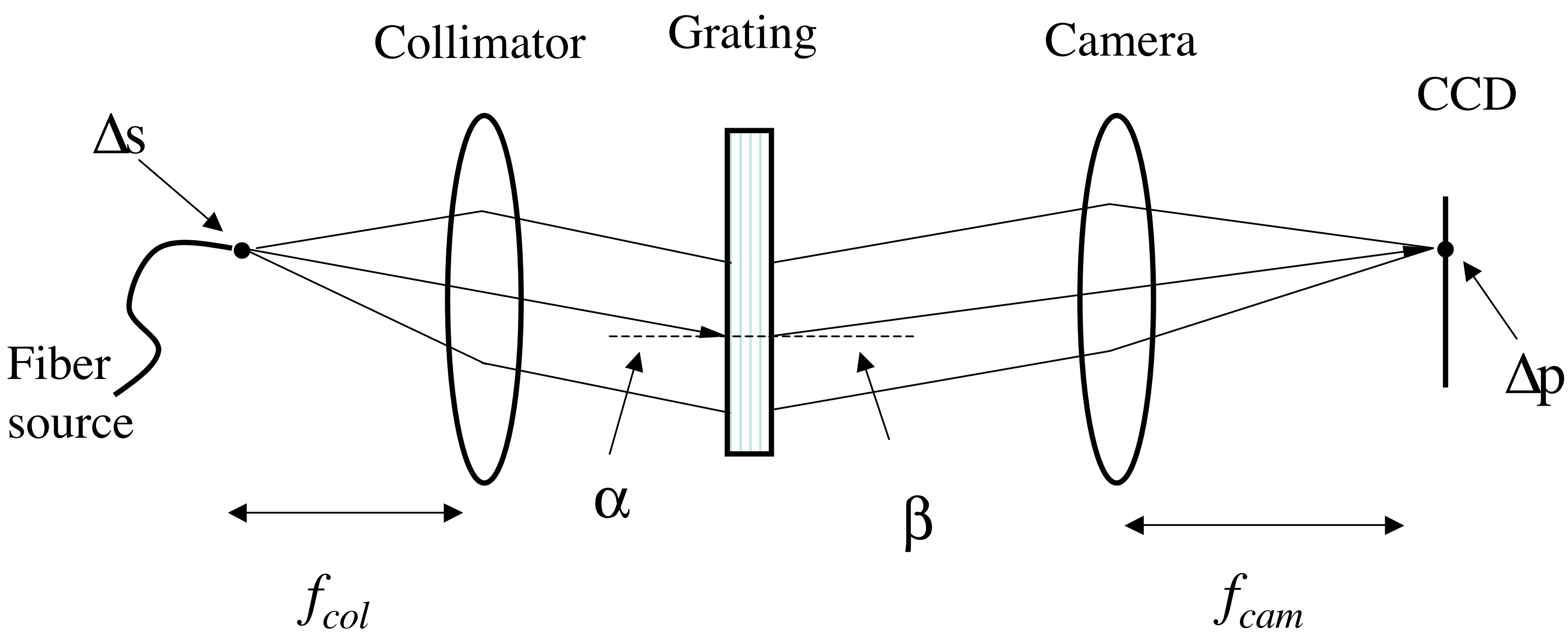
$$R = \frac{\lambda}{\delta\lambda} = \frac{(\sin\alpha + \sin\beta)}{\cos\beta}\frac{D_{cam}}{\lambda} \approx 2\frac{D_{cam}}{\lambda}\tan\theta_B$$

Spectral Resolution: the Diffraction Limit

- $D_{cam} = 75$ mm
- $\tan\,\theta_B = 2$
- $\lambda = 0.632\,\mu\text{m}$

$$R_{DL} \approx 470{,}000$$

Slit Limited Spectral Resolution



$$\Delta\alpha = \Delta s / f_{col}, \quad \Delta\beta = \left(\frac{\partial\beta}{\partial\alpha}\right)_{\lambda} \Delta\alpha, \quad \Delta p = f_{cam}\Delta\beta$$

Slit Limited Spectral Resolution

- Generally, the source is not a point
  - If the extent is greater than the diffraction blur then the spectrometer resolution "slit limited"

$$\delta\alpha = \frac{\delta s}{f_{col}}$$

$$\delta\beta = \left(\frac{\partial\beta}{\partial\alpha}\right)_{\lambda,m}\delta\alpha = \frac{\cos\alpha}{\cos\beta}\delta\alpha = \frac{\cos\alpha}{\cos\beta}\frac{\delta s}{f_{col}}$$

$$\delta\beta = \left(\frac{\partial\beta}{\partial\lambda}\right)_{\alpha,m}\delta\lambda = \frac{m}{\sigma\cos\beta}\delta\lambda$$

$$\frac{m}{\sigma\cos\beta}\delta\lambda = \frac{\cos\alpha}{\cos\beta}\frac{\delta s}{f_{col}} \quad \text{Hence, } R_{SL} = \frac{\lambda}{\delta\lambda} = \frac{\sin\alpha + \sin\beta}{\cos\alpha}\frac{f_{col}}{\delta s}$$

Which is bigger R_{DL} or R_{SL}?