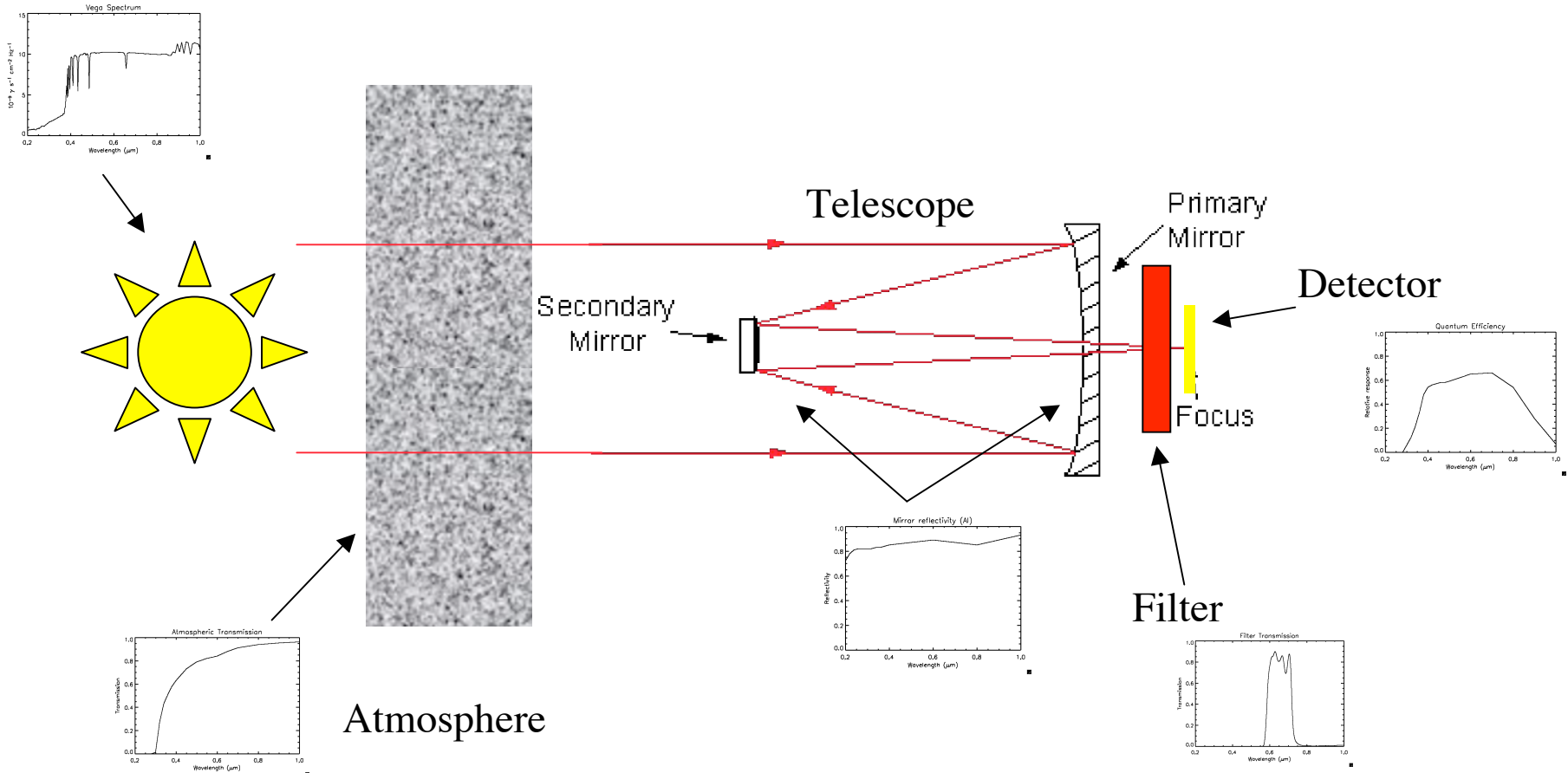


From Stars to Photoelectrons

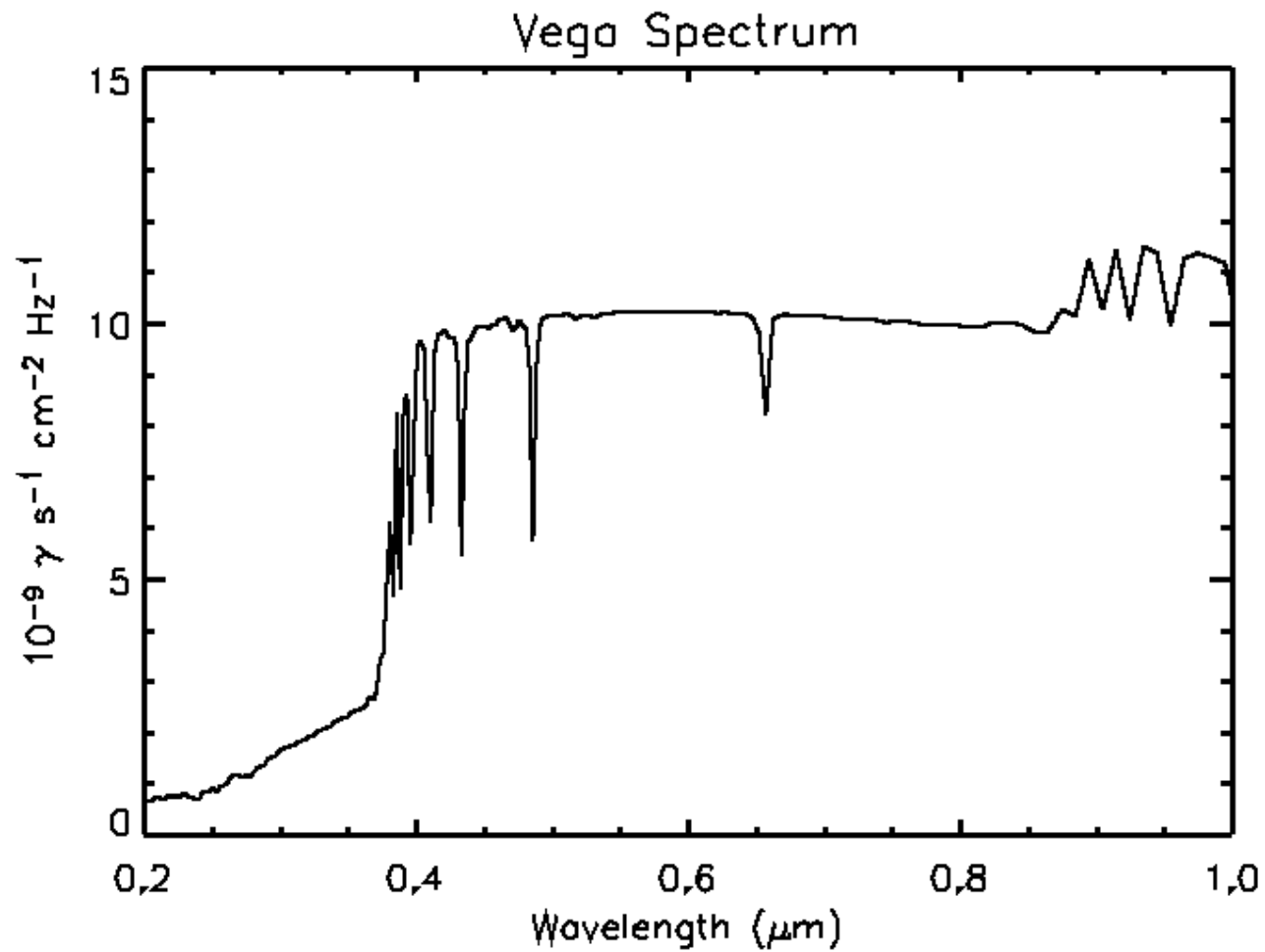
James R. Graham

10/6/2008

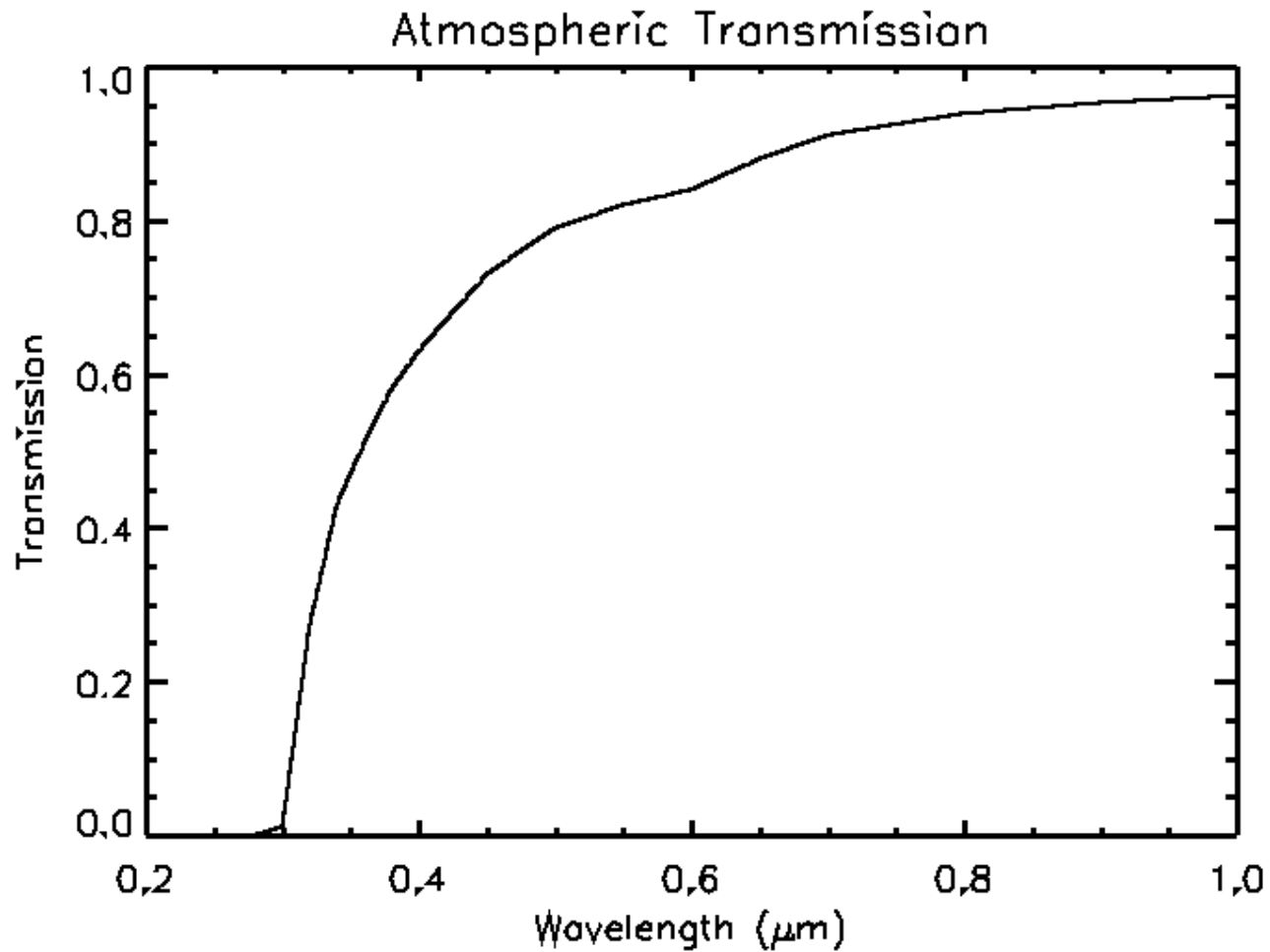
Step 1: The Photon Path



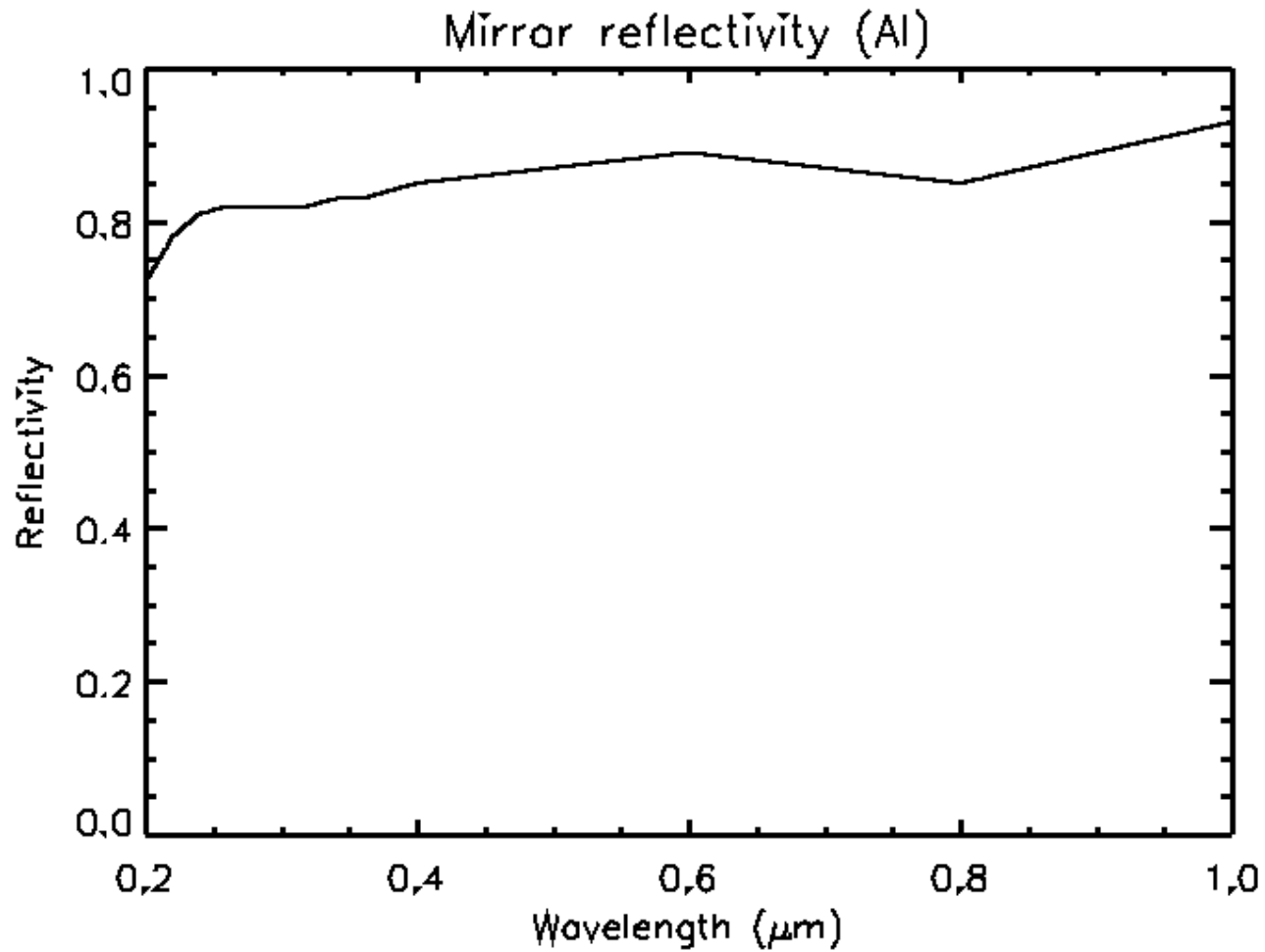
Spectrum of Vega



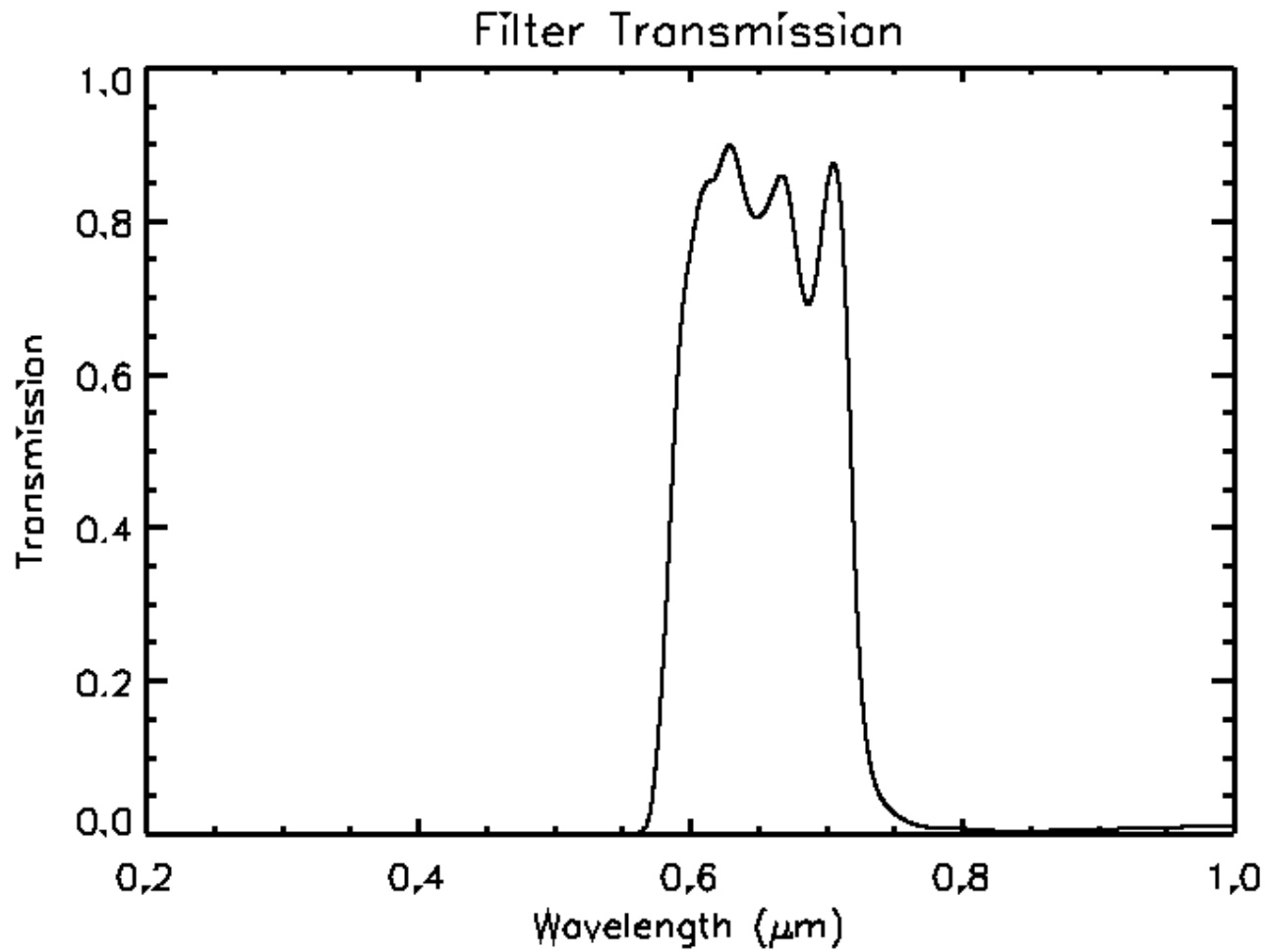
Scattering & Absorption by the Earth's Atmosphere



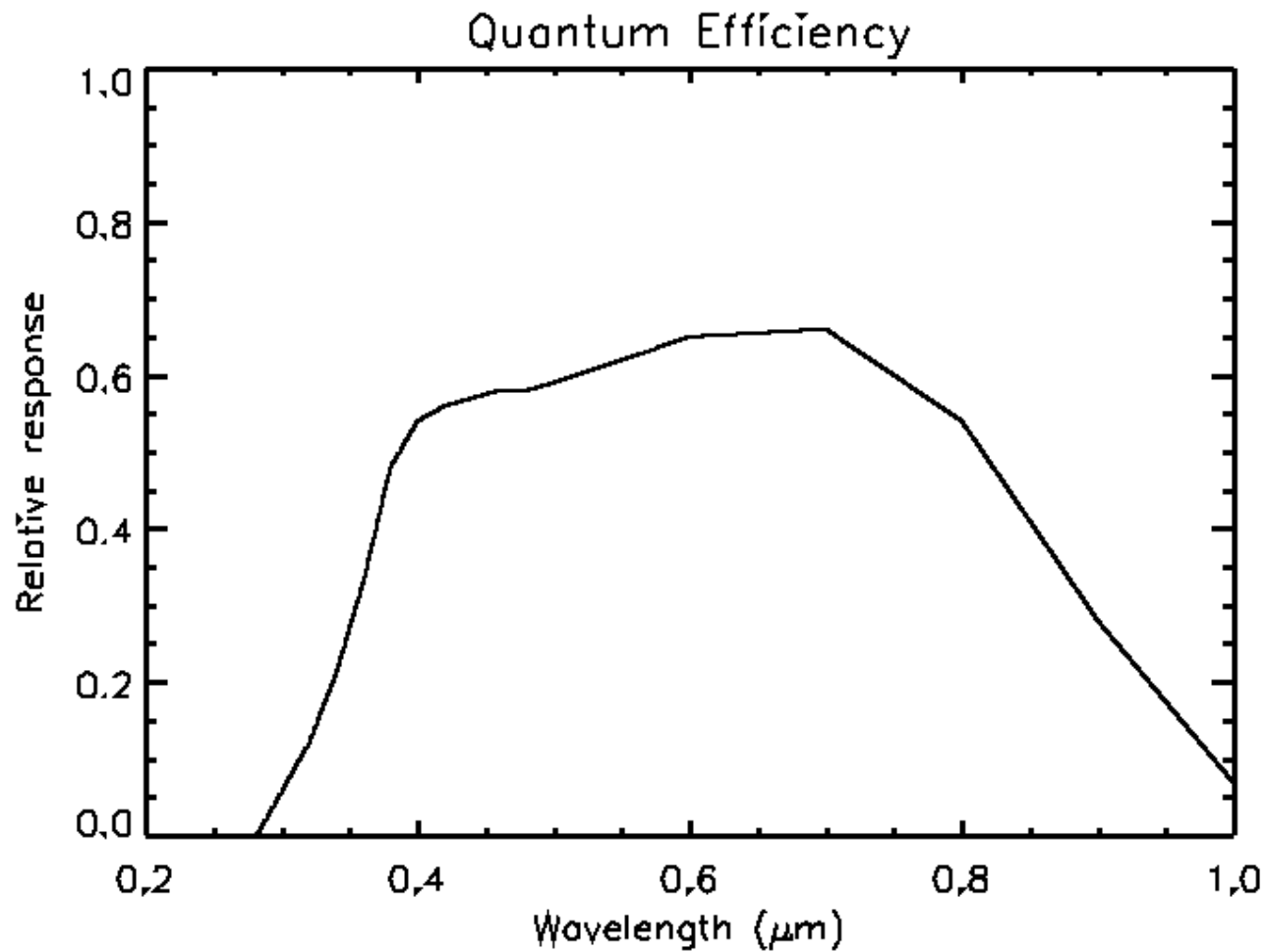
Mirror Reflectivity



Filter Transmission

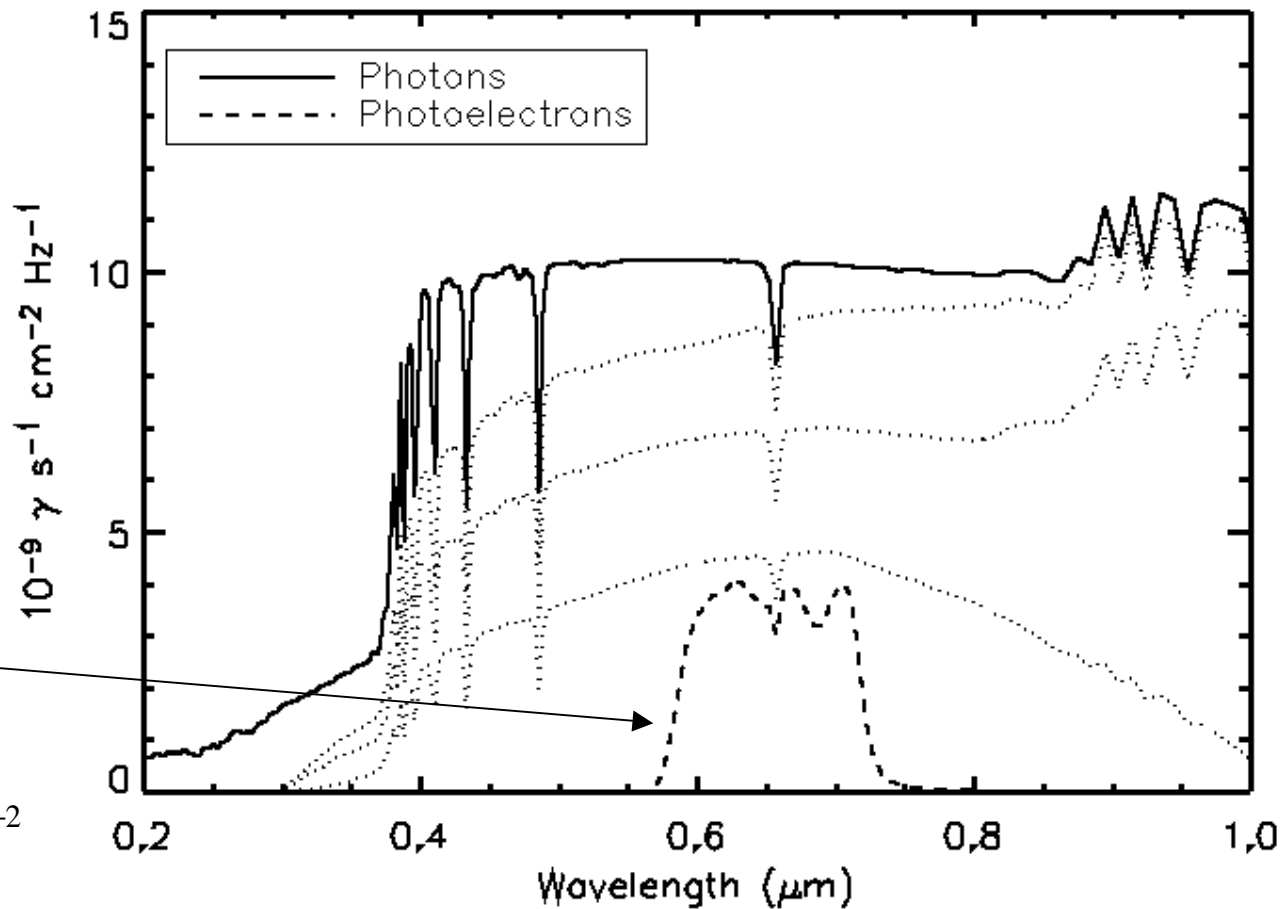


Detector Efficiency



System Throughput

Photoelectron rate



$$\int_{\nu_1}^{\nu_2} \frac{\eta_\nu F_\nu}{h\nu} d\nu =$$

$$5.88 \times 10^{10} \gamma \text{ s}^{-1} \text{ cm}^{-2}$$



Step 2: Systematic Errors

- Imaging detectors suffer from a number of errors that must be corrected before the data can be used for photometry
- Goal is to make the DNs from the FITS files proportional to the brightness of the astronomical source

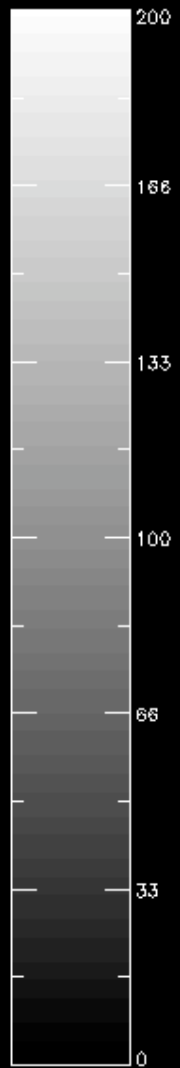
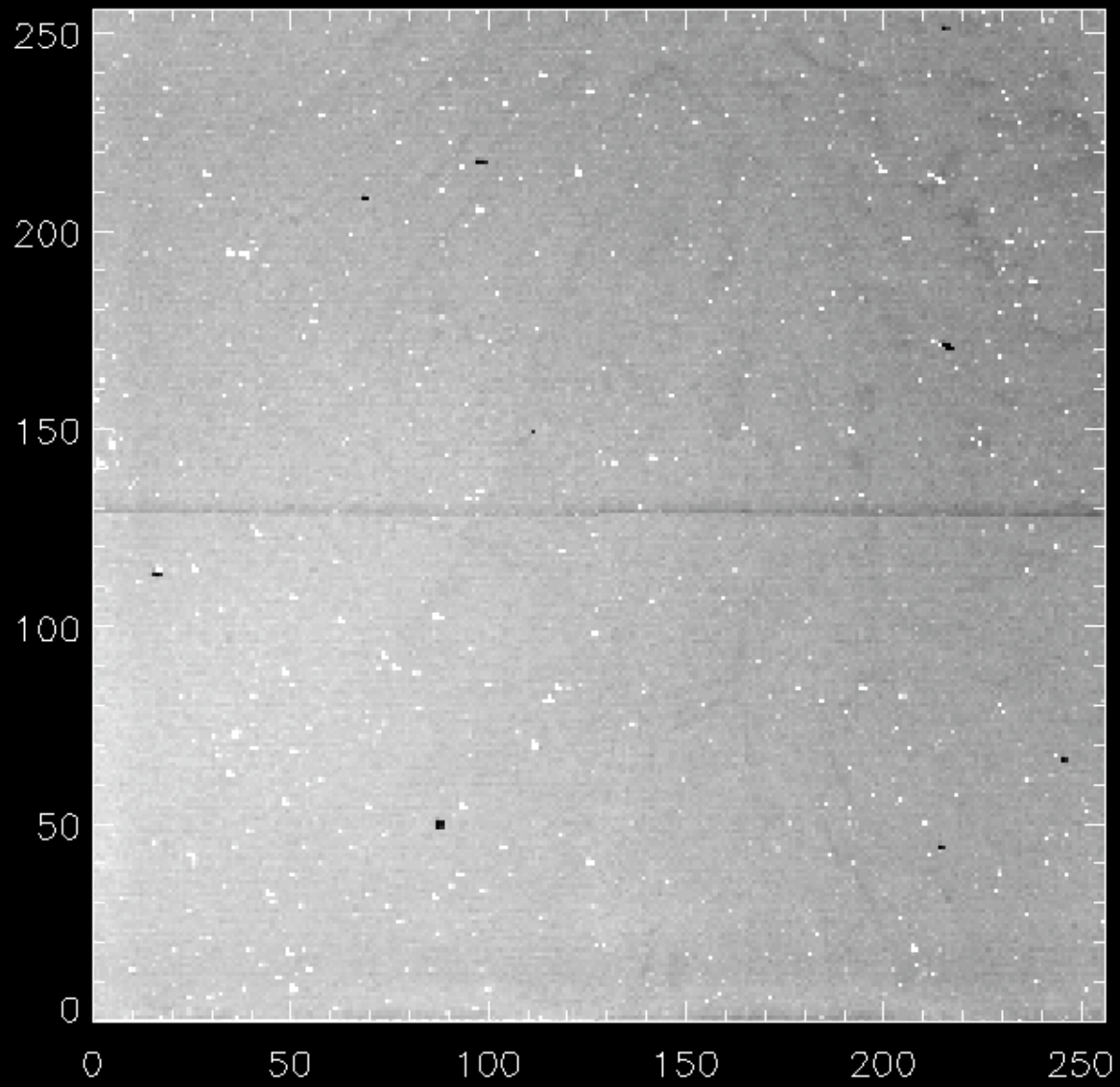
Bias & Dark Current

- Even a zero second exposure gives non-zero DN
 - Dark current masquerades as real signal
 - Dark current & bias (constants DC offset) can be removed either by subtracting
 1. A dark frame of the same exposure time as the science image—takes care of bias too, or
 2. An image of blank sky—takes care of bias & dark, and also subtracts the sky brightness! (can be hard to find blank sky)

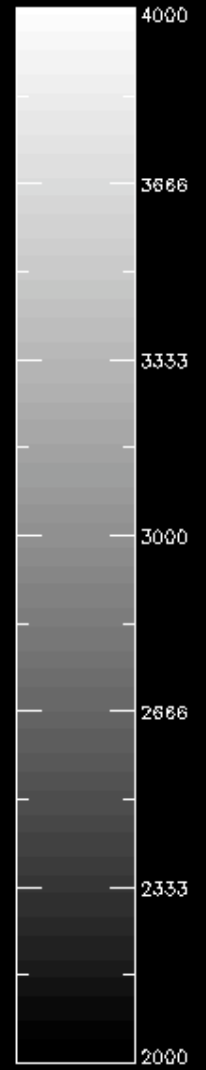
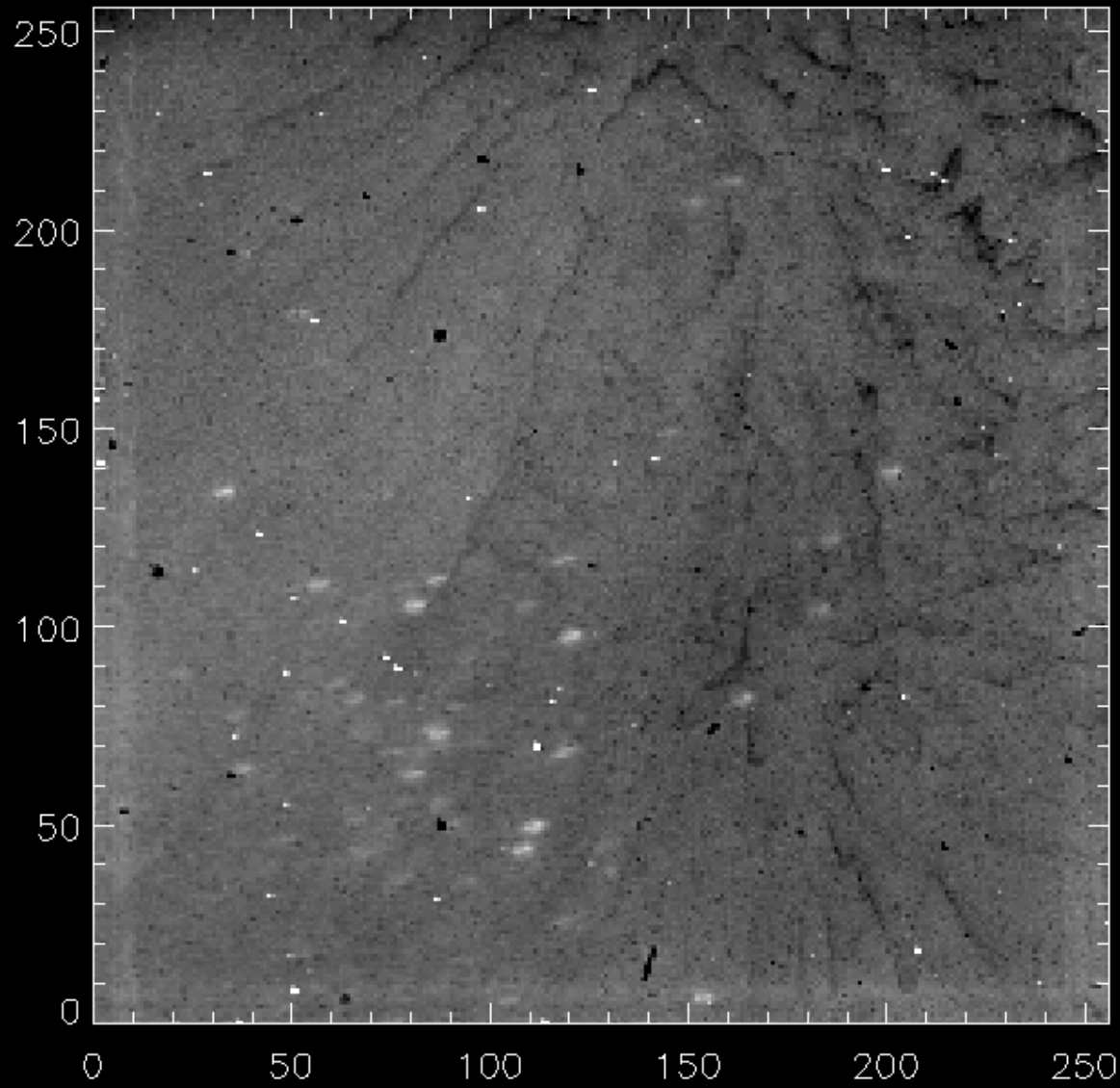
Relative Pixel Gain *a.k.a.* Flat Field

- Every pixel in the detector array has a slightly different response to light
 - Some pixels are more efficient than others
- Need to correct for pixel-to-pixel variations by constructing a flat field
 - Make a flat field by observing a uniform source, e.g., the twilight sky
 - *Divide dark-subtracted images by the flat field*

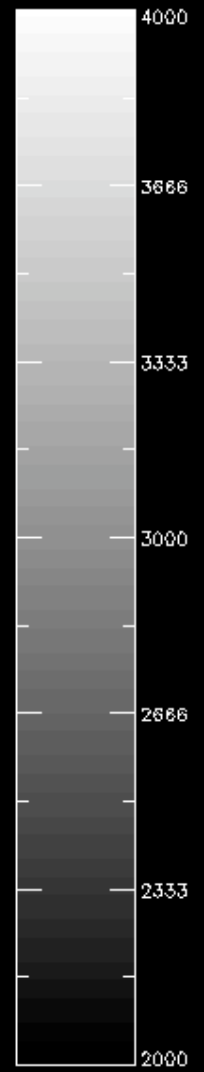
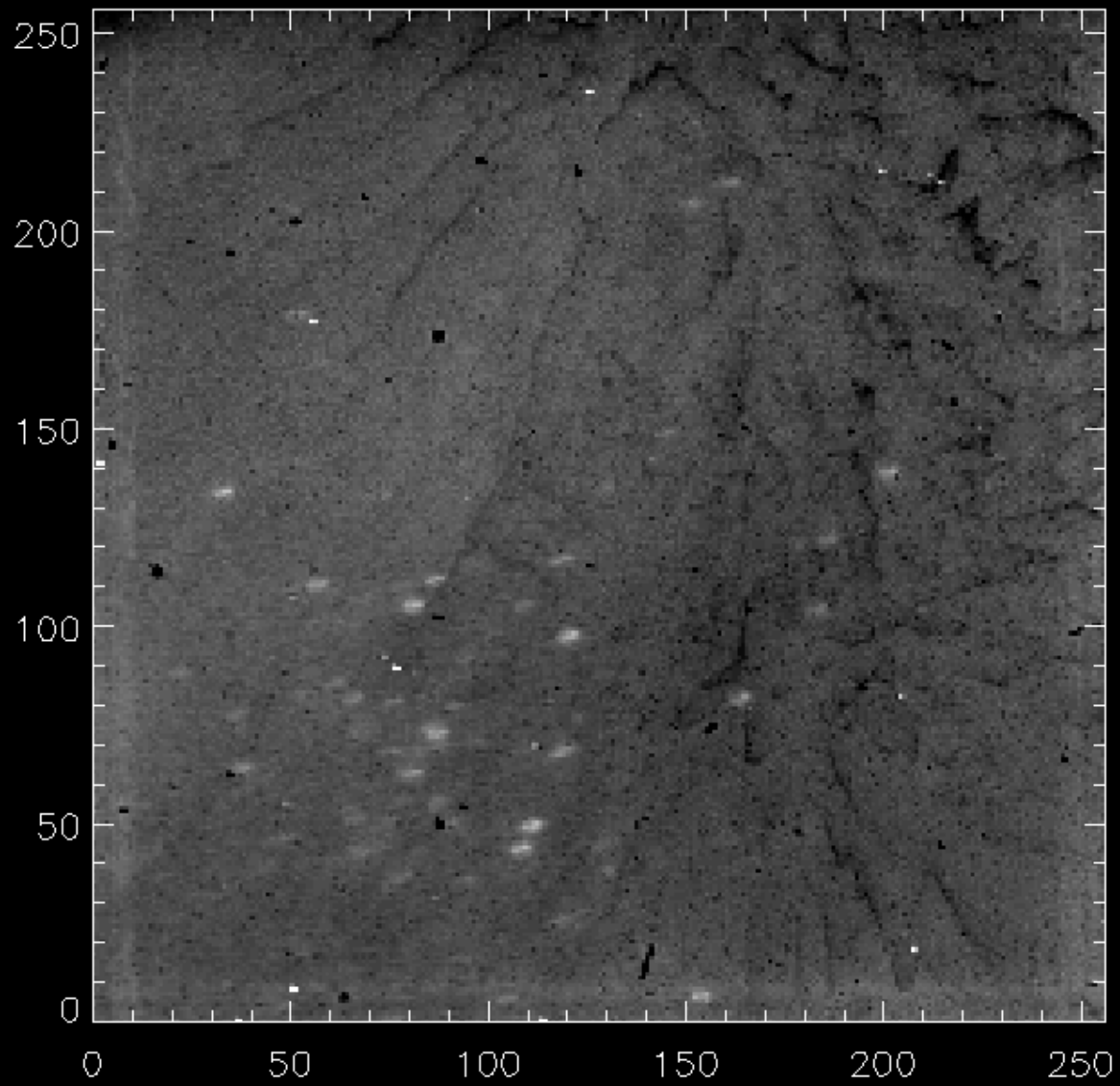
Dark



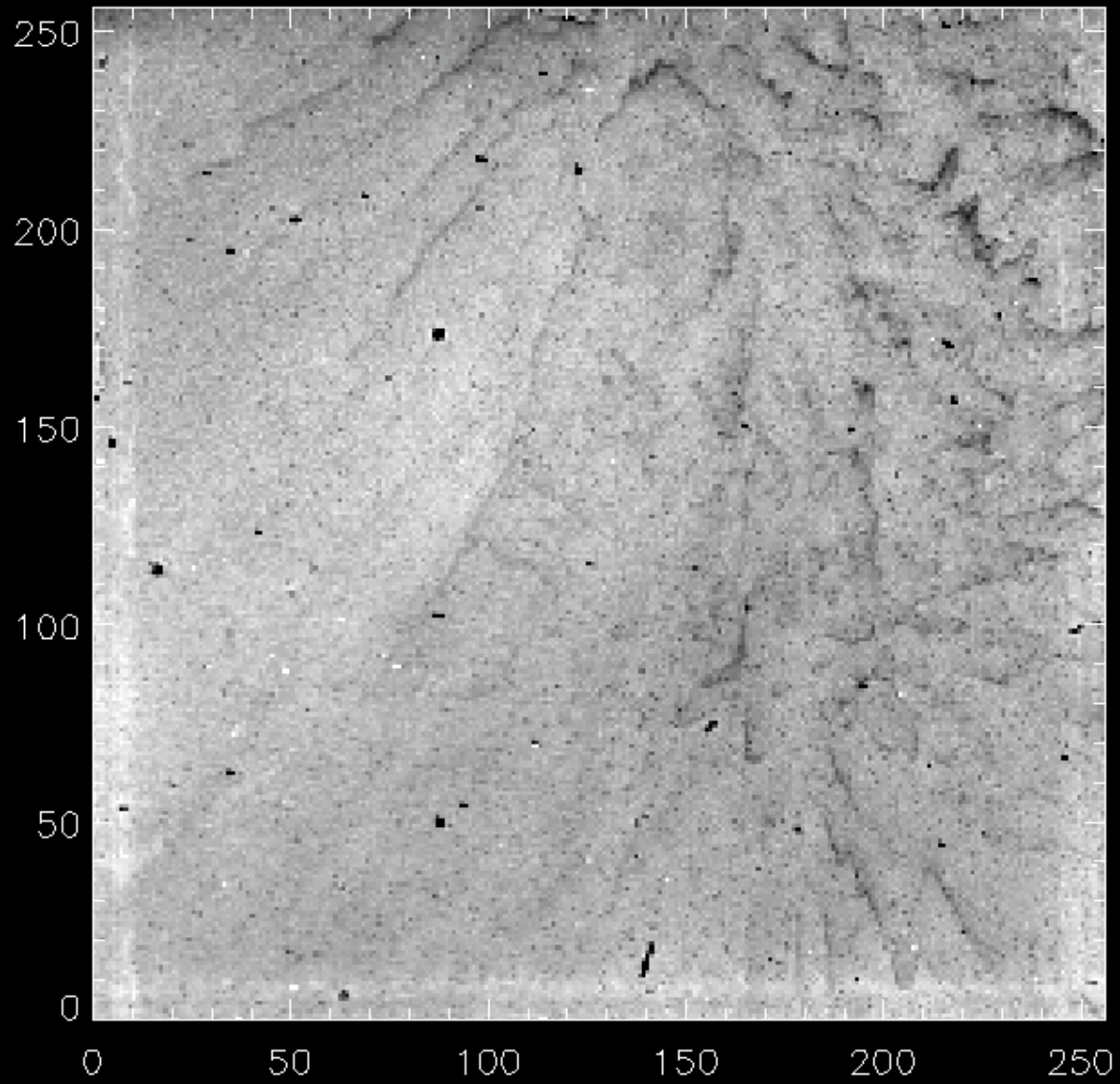
M13 Raw



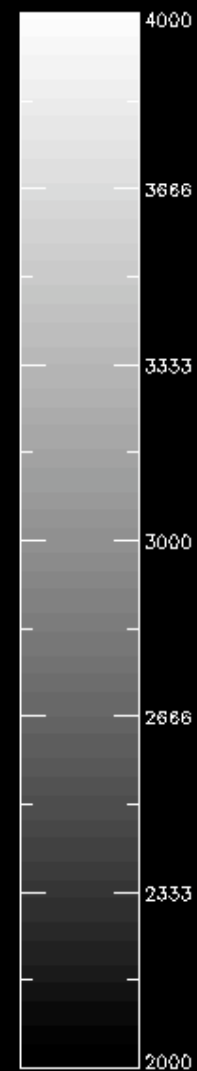
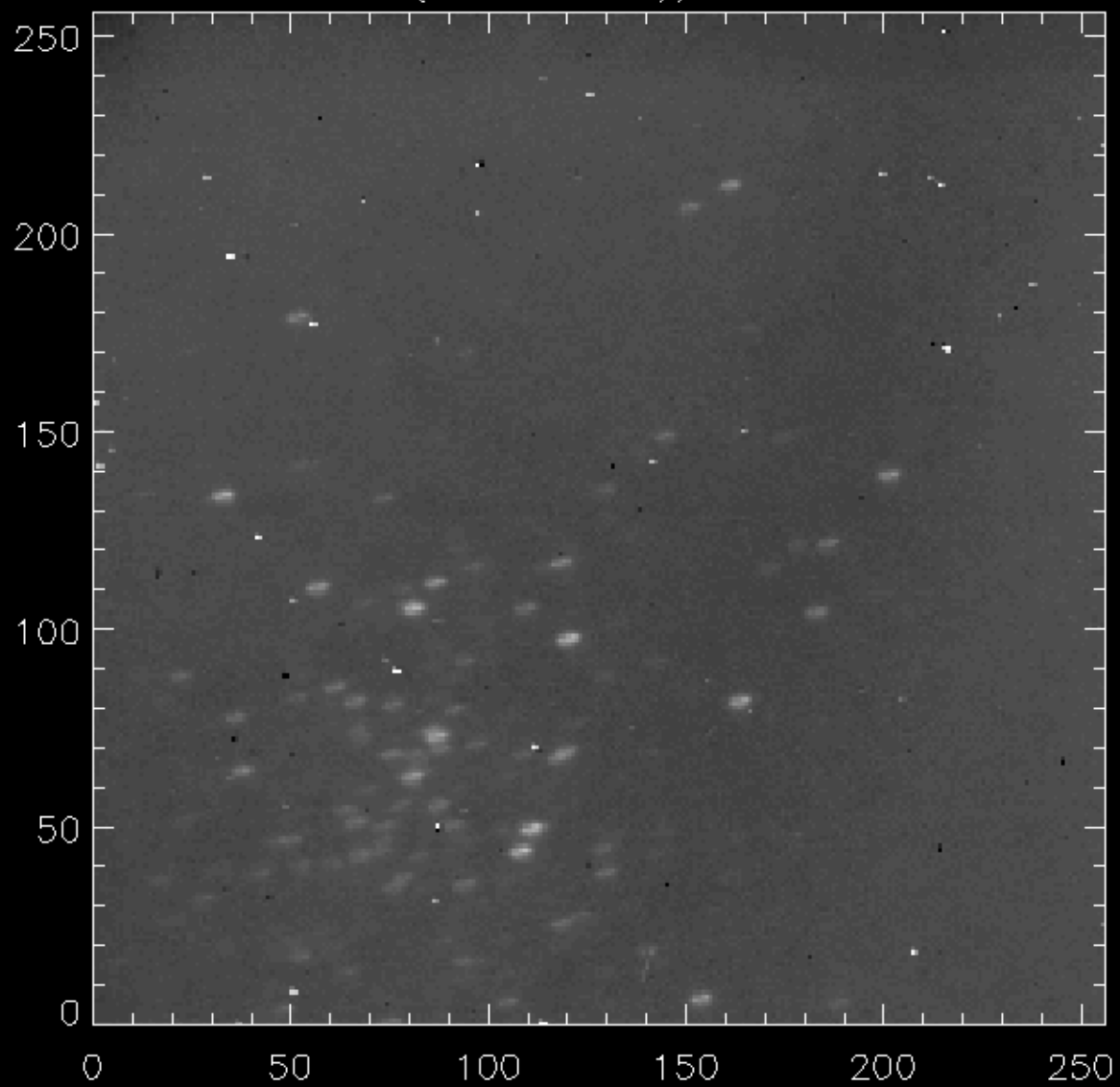
M13-Dark



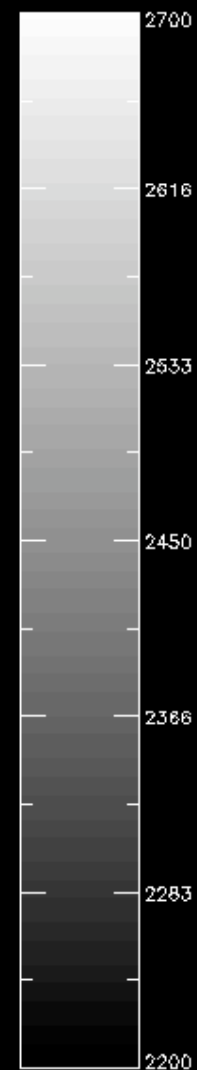
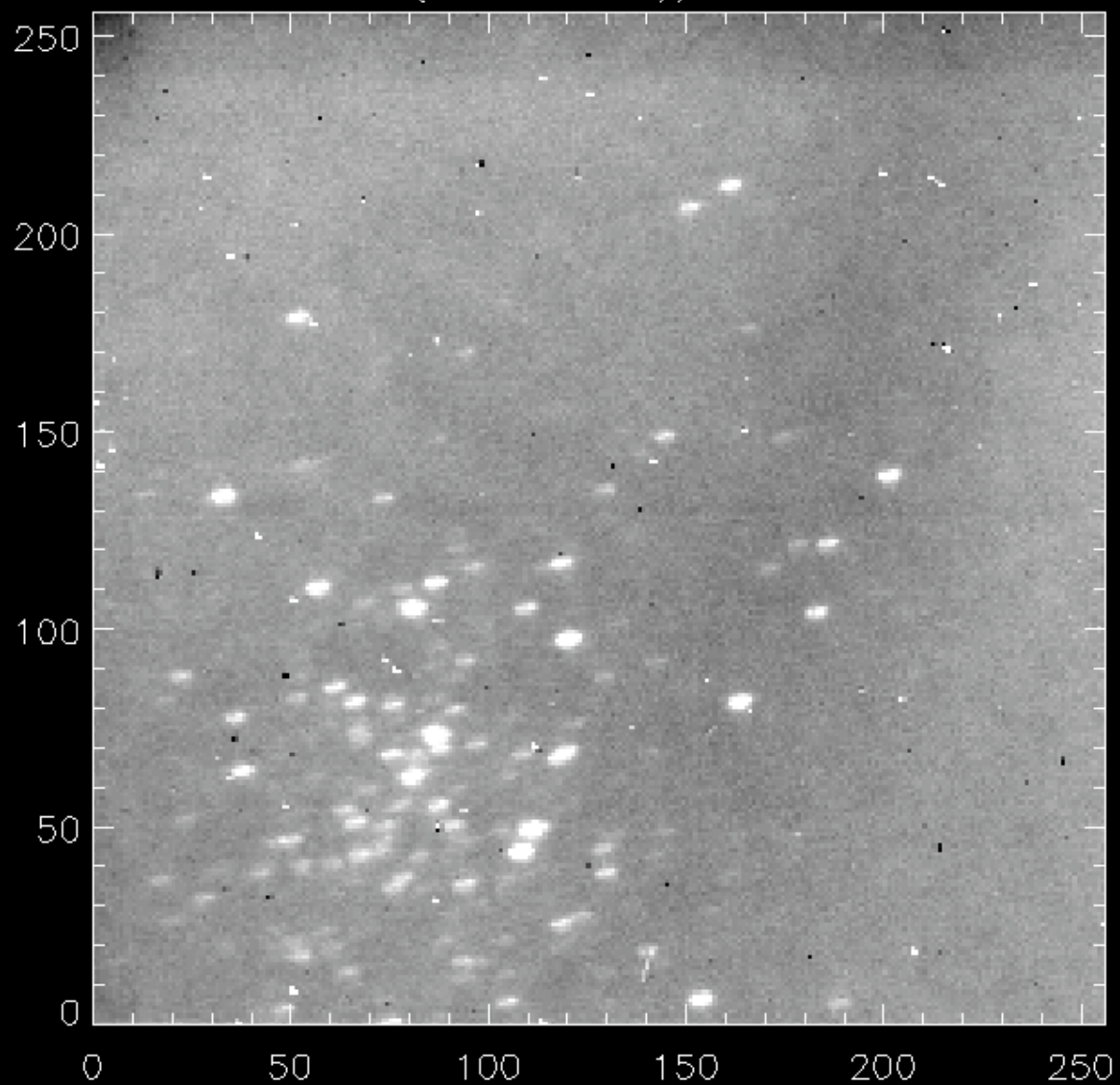
Flat



(M13-Dark)/Flat

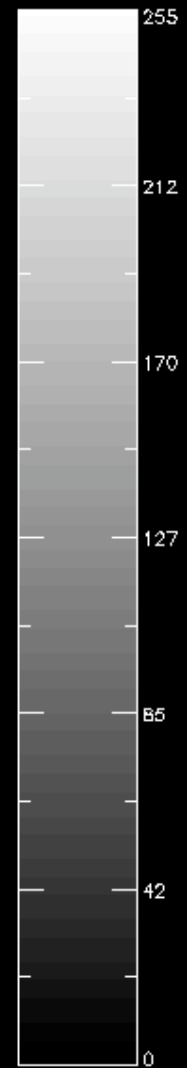
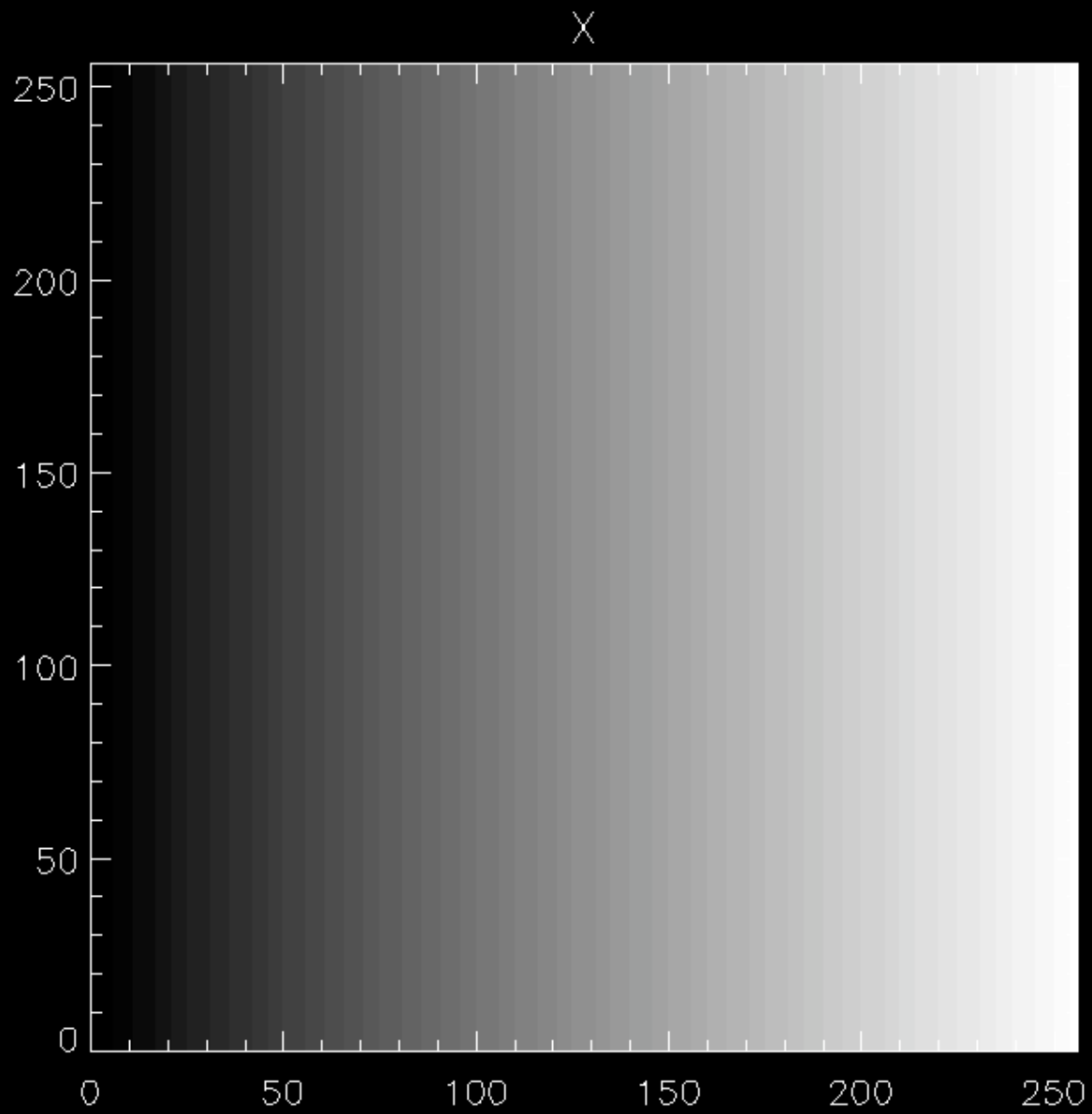


(M13-Dark)/Flat

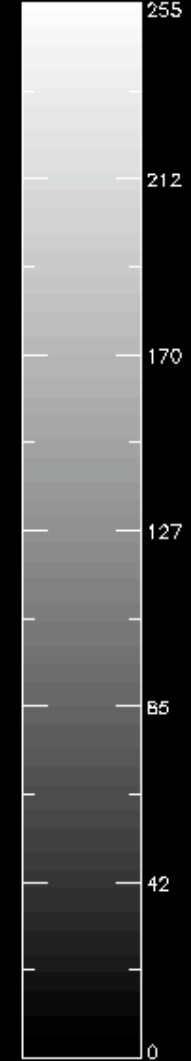
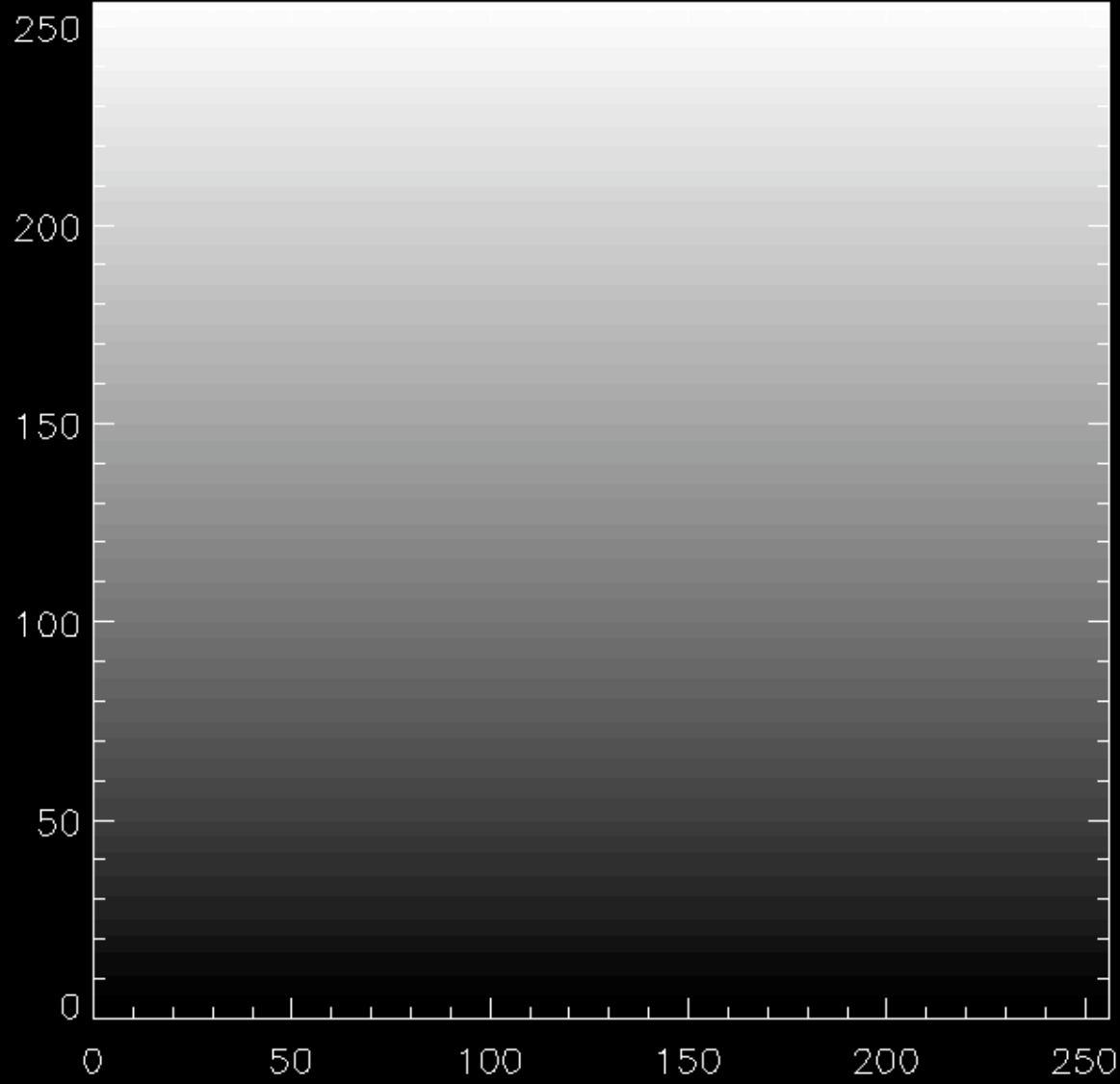


How Bright is that Star?

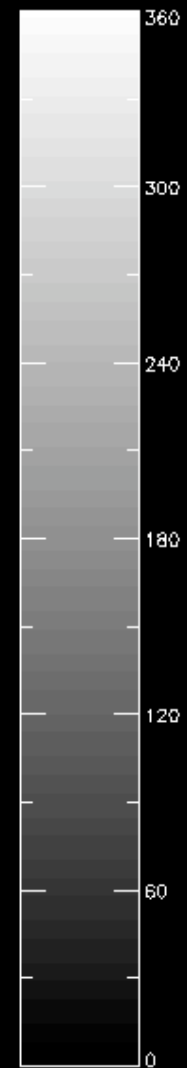
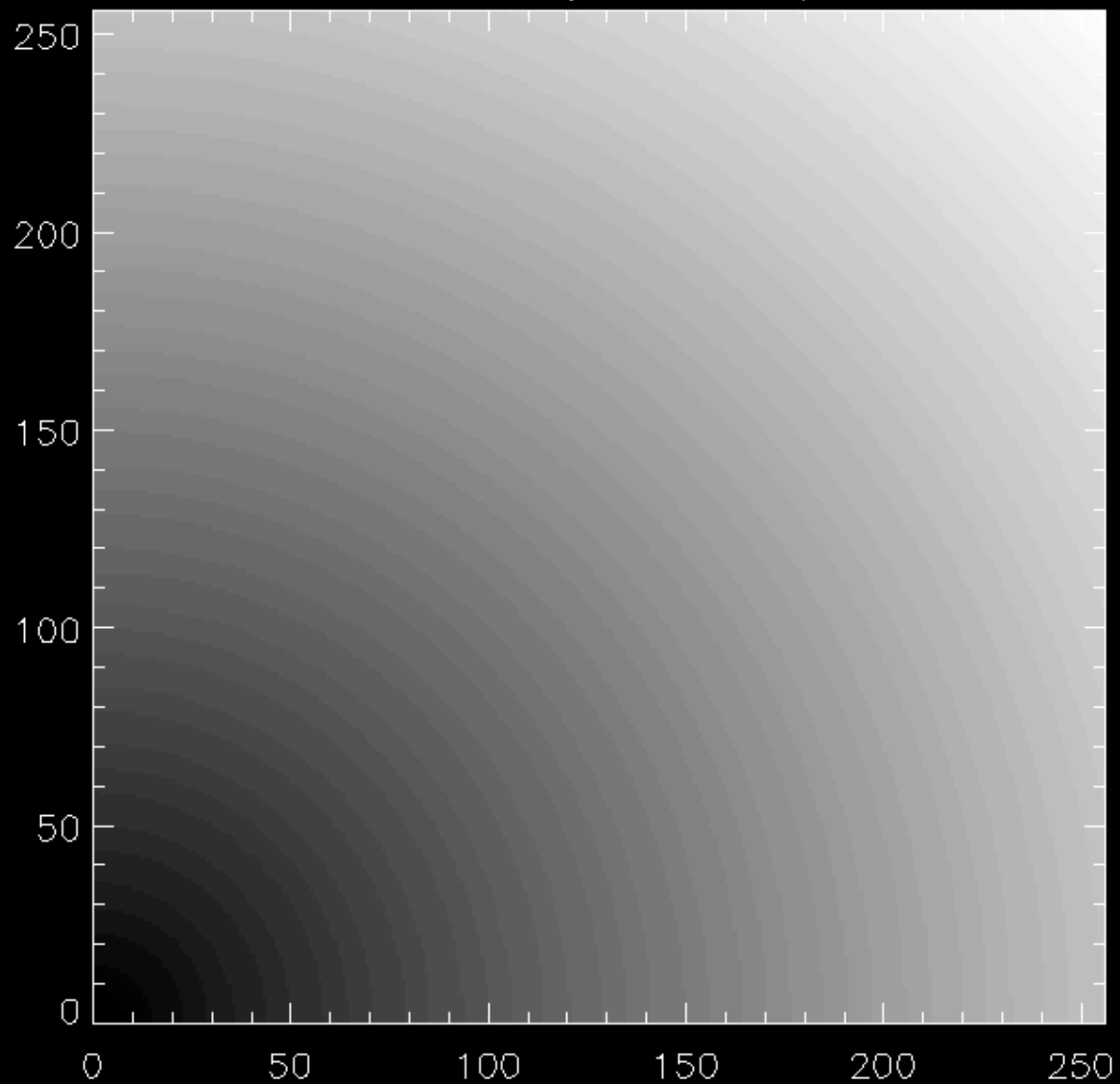
- The light from a star is spread over several pixels
- How do we sum the light to get a measure of the total signal from the star?
 1. Identify the location of the star (**RDPIX**)
 2. Select the associated pixels by making a mask
 3. Sum up the light (**TOTAL**)
 - Subtract the sky background if necessary



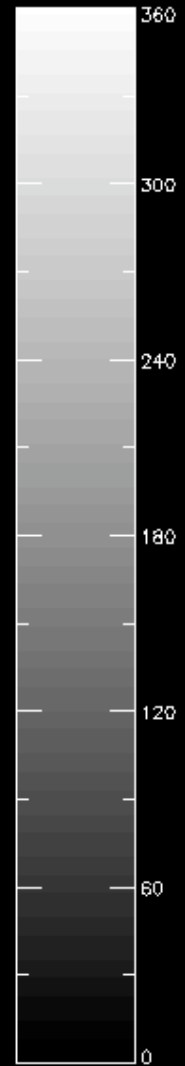
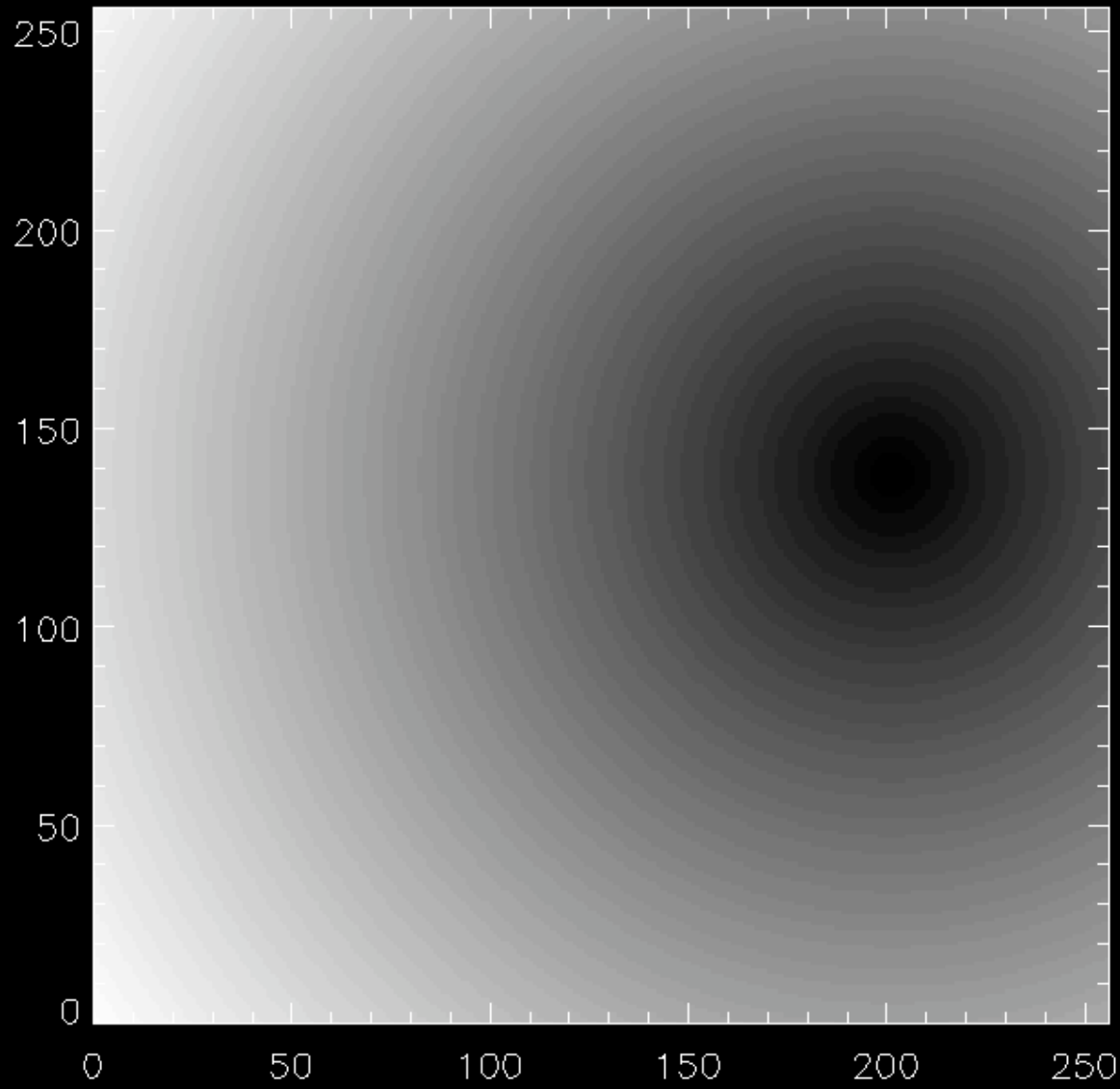
Y



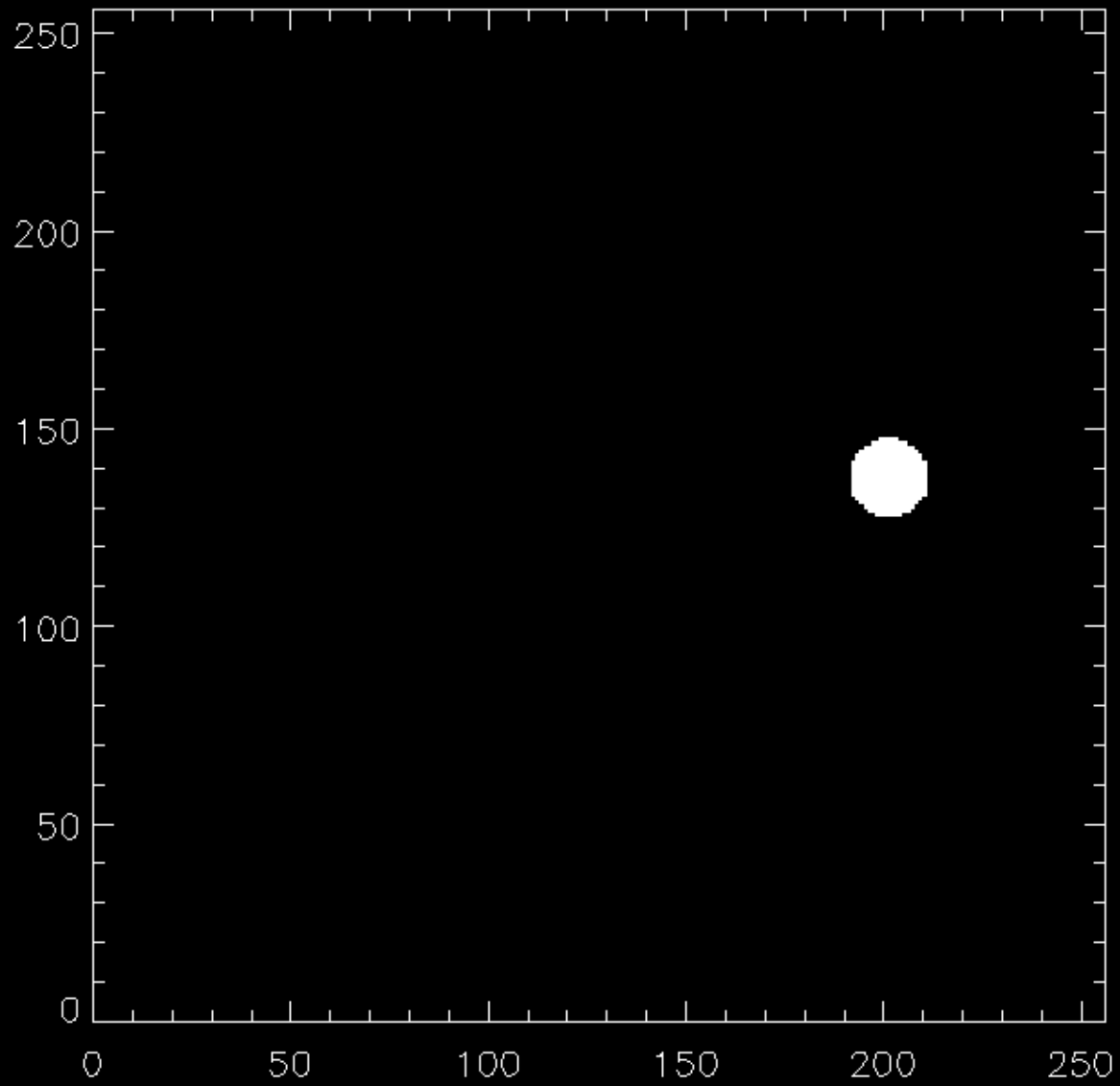
$$R = \text{SQRT}(X^2 + Y^2)$$



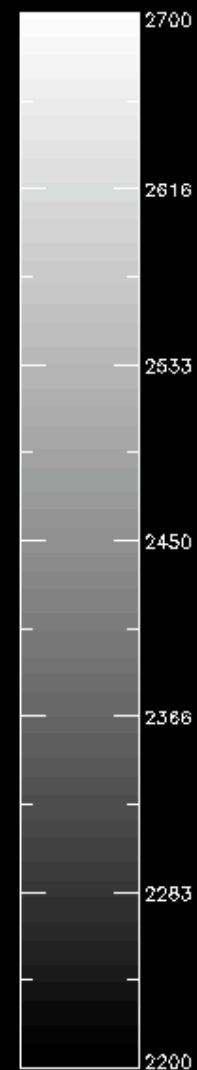
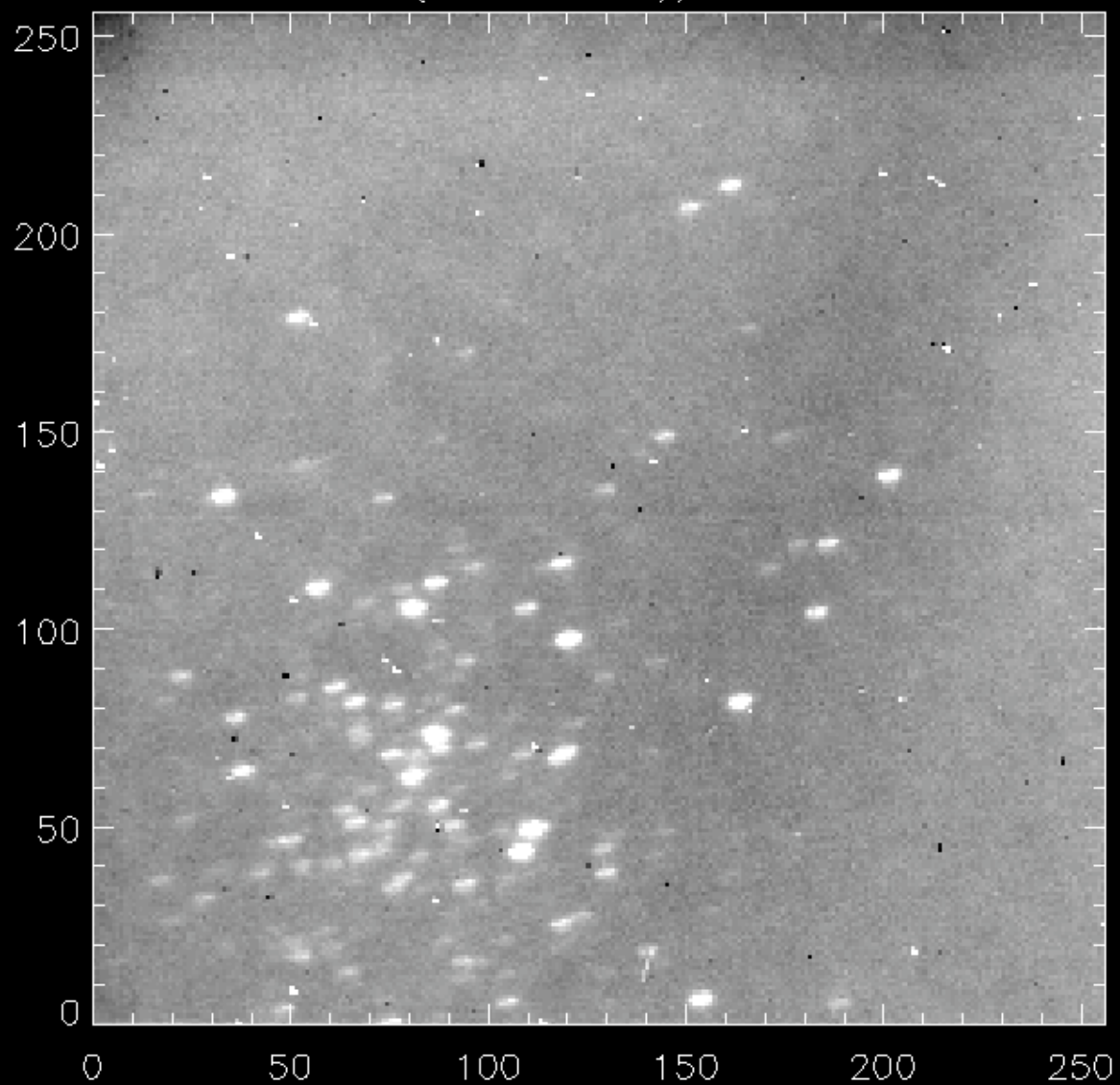
$$R = \text{SQRT}((X - X_0)^2 + (Y - Y_0)^2)$$



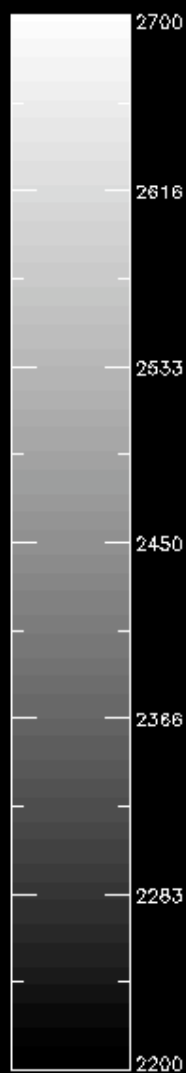
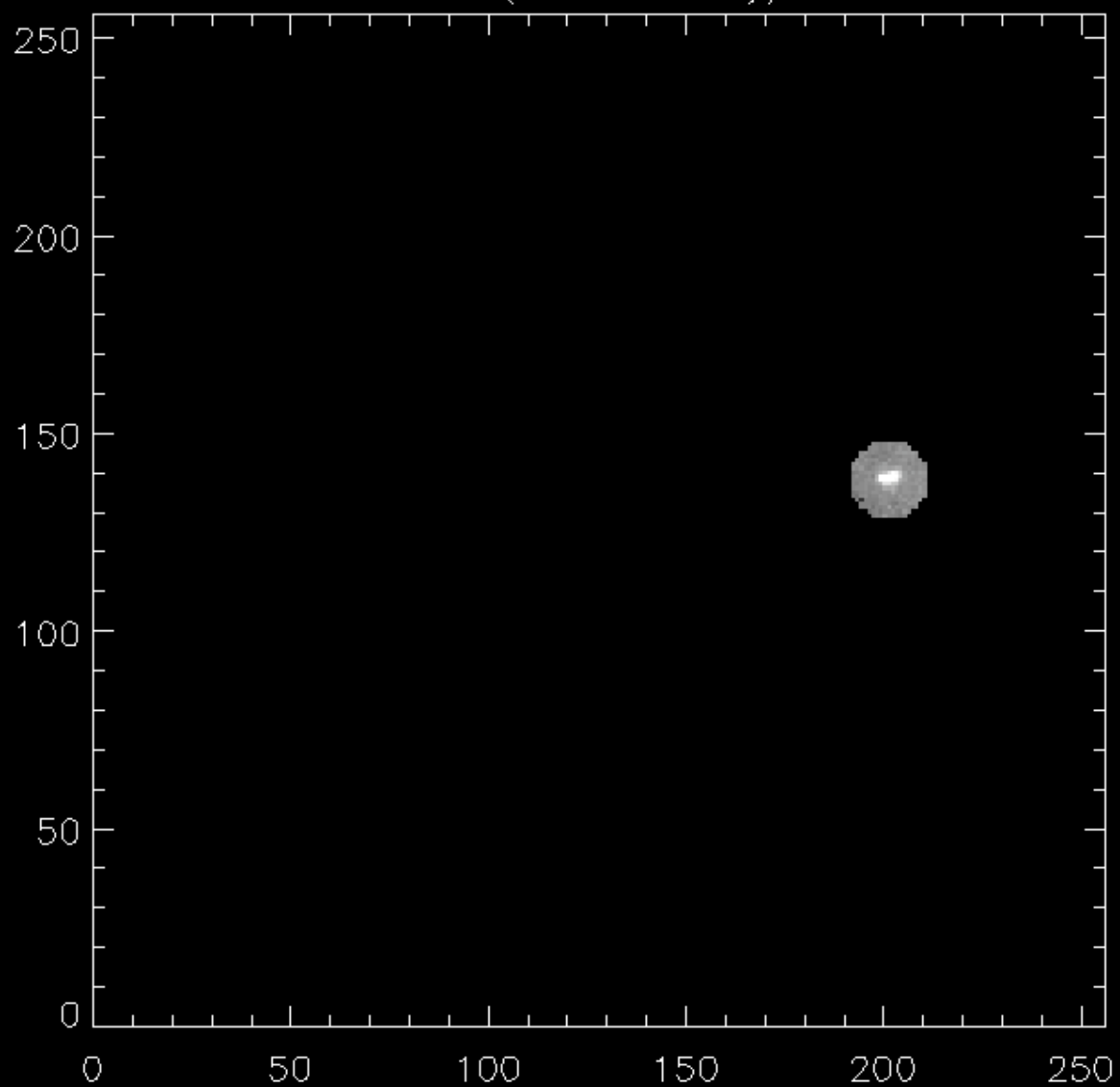
Mask



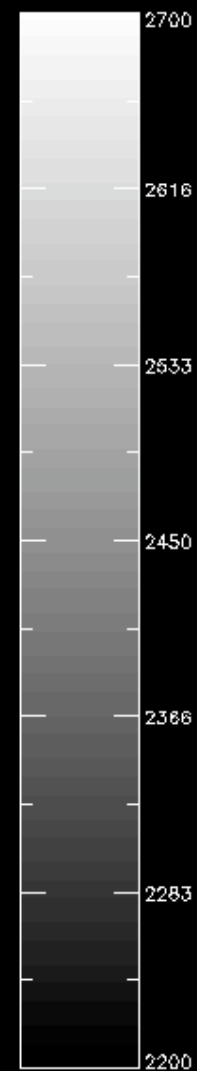
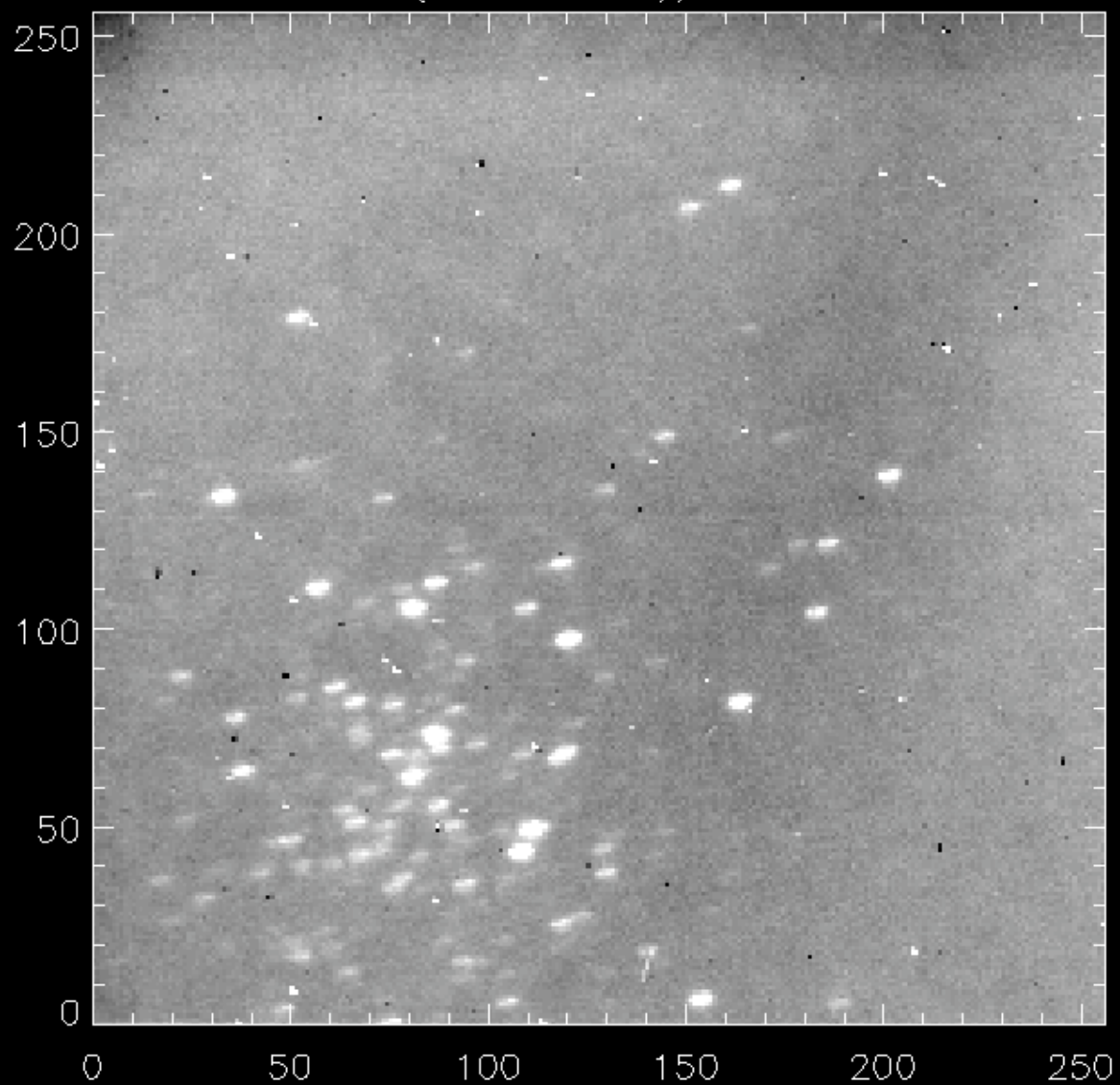
(M13-Dark)/Flat



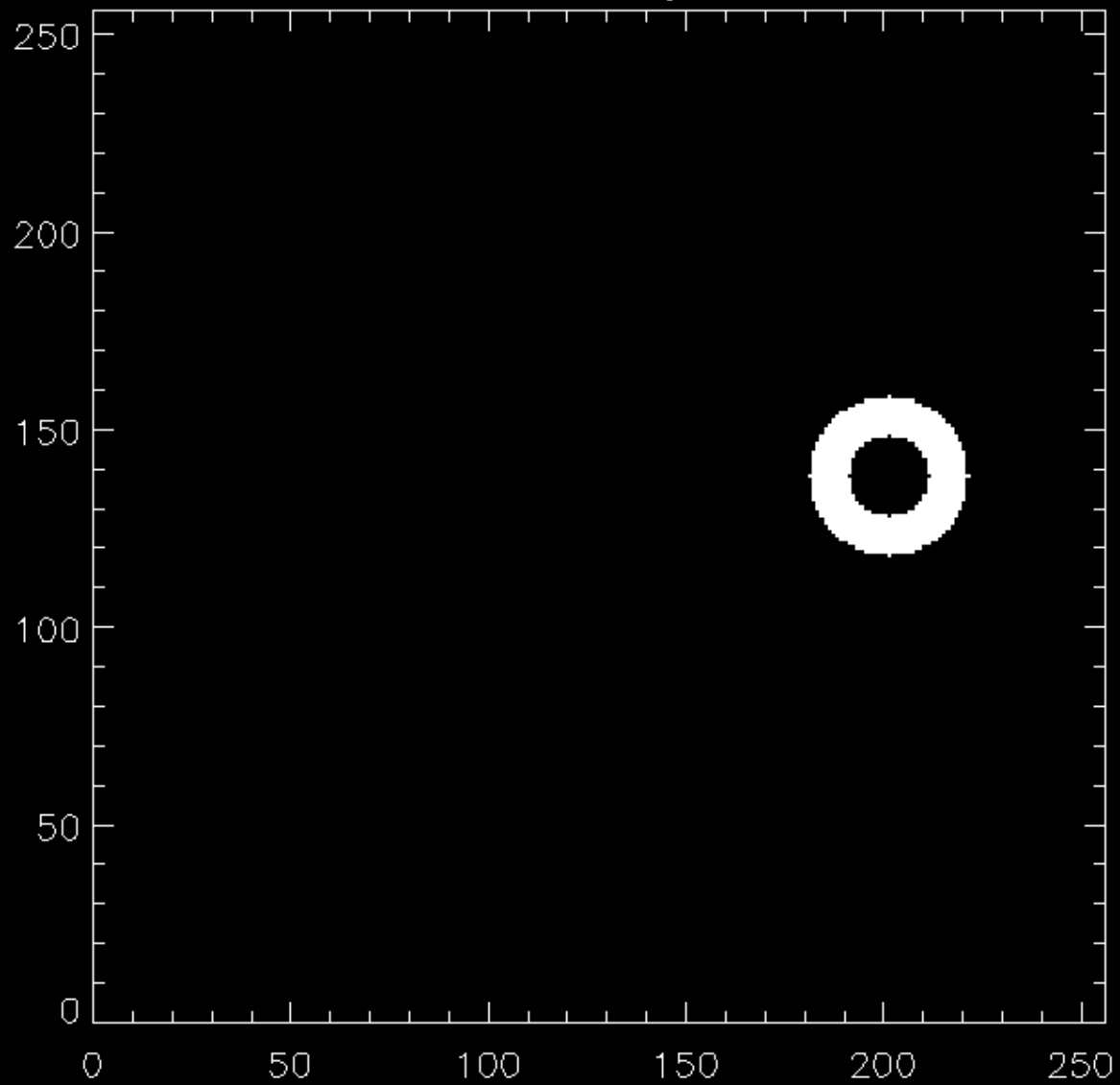
Mask x (M13-Dark)/Flat



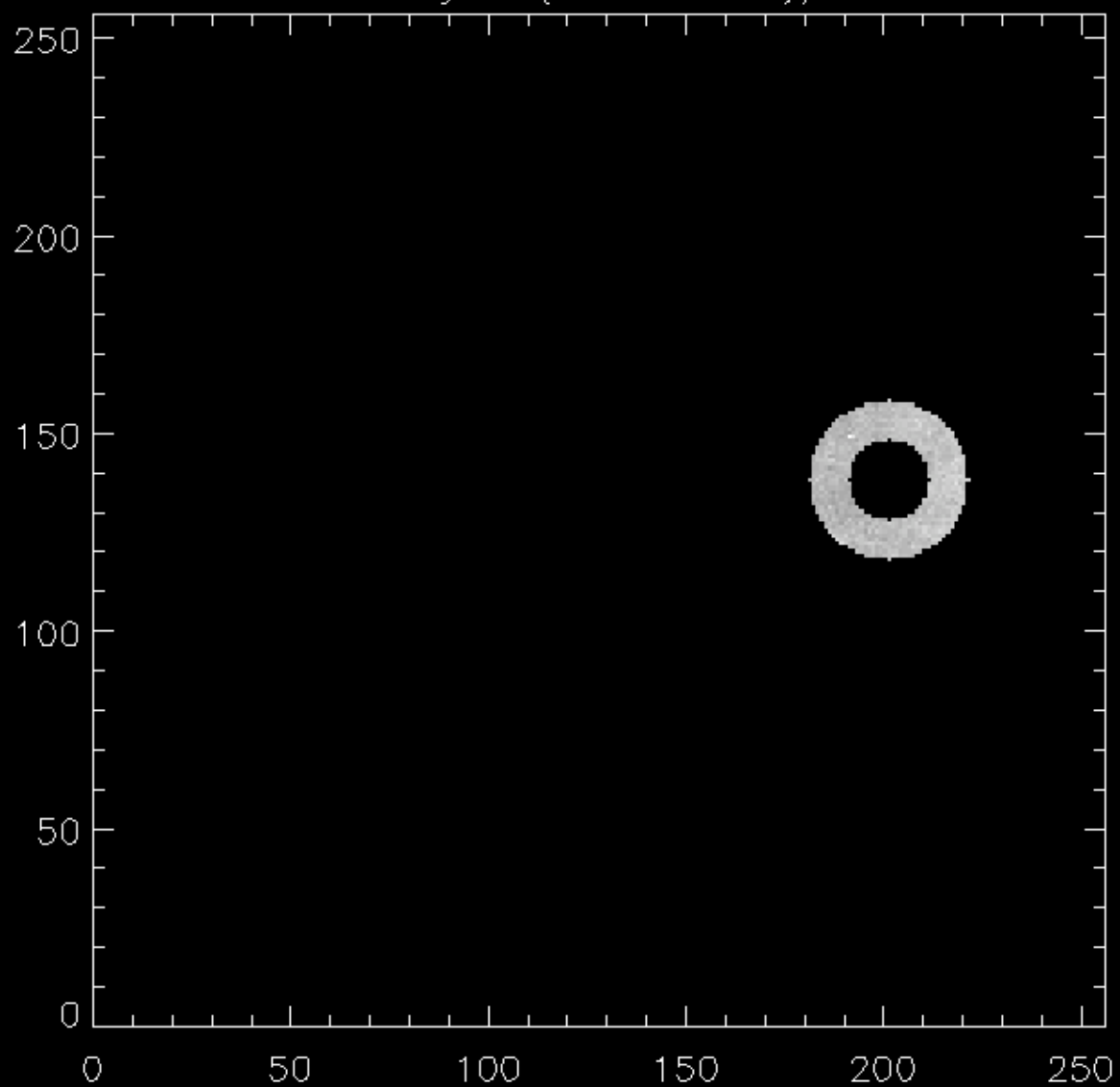
(M13-Dark)/Flat



Masksky



Masksky x (M13-Dark)/Flat



Computing the Centroid

```
skyval = median( px[wsky] )
print, 'Median sky value = ',skyval

; compute the pixel centroids

xbar = total(mask*xx * (px-skyval) )/total(mask*(px-
skyval))
ybar = total(mask*yy * (px-skyval))/total(mask*(px-
skyval))

print, '<x> = ', xbar
print, '<y> = ', ybar
```

Step 3: Modeling the Noise

- What is the SNR of a given observation?
- How do I choose and optimize the photometric parameters
 - Exposure time required?
 - Aperture diameter?
 - Location and size of sky annuli?

How to Begin

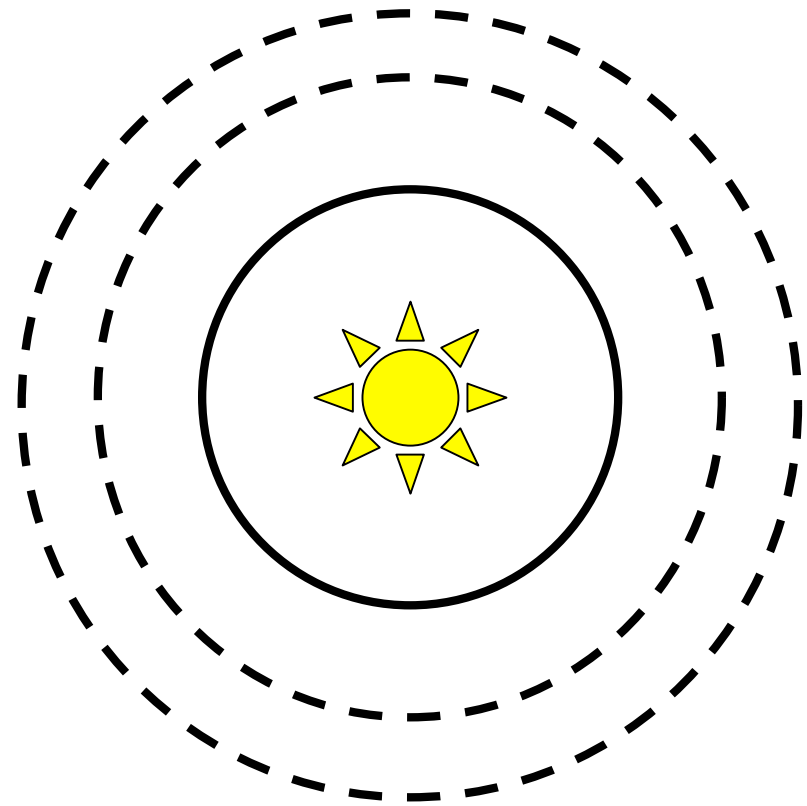
- Write down an expression for the signal and use error propagation to find the noise
 - Express results as signal-to-noise ratio vs. photometric parameters

The Model

- The purpose is to *estimate* the noise contributions
 - Often getting the answer to within a factor of two is fine
 - Make simplifying assumptions — so long as you can justify them

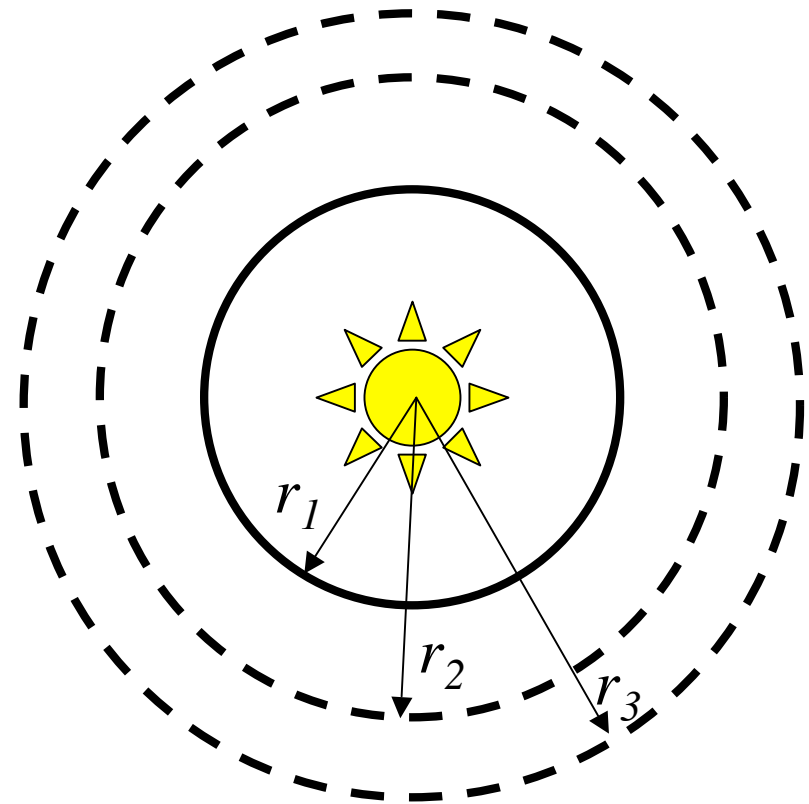
A Photometric Model

- What parameters describe the measurement?



A Photometric Model

- Star
 - Brightness
 - Center (x_0, y_0)
 - Width (σ)
- Sky background in annulus
 - B
- Detector
 - QE , readnoise, dark current
- Aperture sizes
 - r_1, r_2, r_3



Photometric Model

- Write down an expression for the signal, S_i , in units of photoelectrons
 - In an individual pixel

$$S_i = F_i + B_i + Q_i + E_i$$

- F_i is the stellar signal = $f_i t$ at pixel i [e^-]
 - Different for every pixel
- Q_i is the dark charge = $i_i t$ [e^-] in a given pixel
 - The dark current i_i varies from pixel to pixel
 - For SNR model assume constant
- B_i is the sky background = $b_i t$ assumed uniform [e^-]
 - Varies from pixel to pixel, for SNR model assume constant
- E_i is the readout electronic offset or bias [e^-]
 - Varies from pixel to pixel, for SNR model assume constant

The Stellar Signal

- The stellar signal is found by subtracting the background from S_i and summing over the N pixels that contain the star

$$F_i = S_i - (B_i + Q_i + E_i)$$

$$F_N = \sum_{i=1}^{N_1} F_i = \sum_{i=1}^{N_1} S_i - (B_i + Q_i + E_i)$$

$$N_1 = \pi r_1^2$$

- Error in F_N is due to noise in the signal itself, F_N
- Noise due to dark charge, Q_i
- Noise from the background, B
- The read out noise σ_{RO}

Noise Sources

$$F_N = \sum_{i=1}^{N_1} F_i = \sum_{i=1}^{N_1} \left[S_i - \underbrace{(B_i + Q_i + E_i)}_{\text{Background}} \right]$$

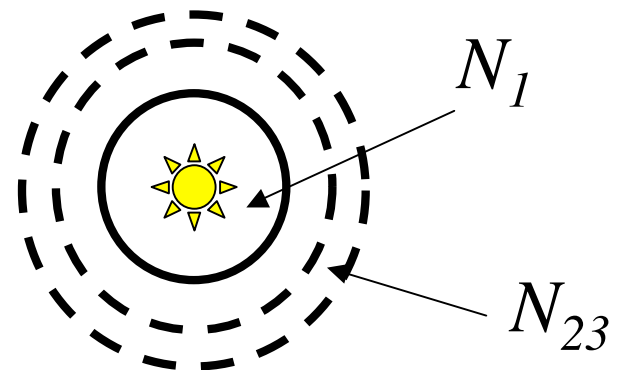
$\langle B \rangle$ = average sky/pixel & $\langle Q \rangle$ the average dark charge/pixel

$$\sigma_F^2 = \underbrace{F_N}_{\text{Poisson signalnoise}} + \underbrace{N_1(\langle B \rangle + \langle Q \rangle + \sigma_{RO}^2)}_{\text{Poisson noise within } r_1} + \underbrace{N_1 \sigma_{Sky}^2}_{\sigma_{Sky} \text{ is the error in the sky measured between } r_2 \text{ \& } r_3}$$

$$\sigma_{Sky}^2 = (\langle B \rangle + \langle Q \rangle + \sigma_{RO}^2) / N_{23}$$

Every pixel between r_2 & r_3 contributes to the accuracy of the sky measurement

$$\underbrace{N_1 = \pi r_1^2}_{\text{Star}}, \quad \underbrace{N_{23} = \pi r_3^2 - \pi r_2^2}_{\text{Sky}}$$



Noise Sources

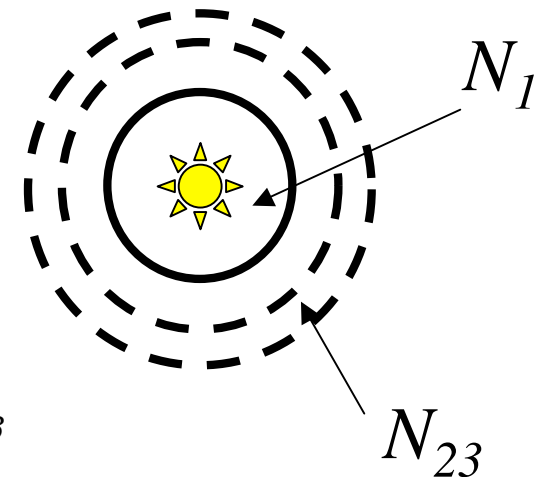
$$F_N = \sum_{i=1}^{N_1} F_i = \sum_{i=1}^{N_1} \left[S_i - \underbrace{(I_i + B_i + E_i)}_{\text{Background}} \right]$$

$\langle B \rangle$ = average sky/pixel & $\langle Q_d \rangle$ the average dark charge/pixel

$$\sigma_F^2 = \underbrace{F_N}_{\text{Poisson signal noise}} + \underbrace{N_1 (\langle B \rangle + \langle Q_d \rangle + \sigma_{RO}^2)}_{\text{Poisson noise within } r_1} + \underbrace{N_1 (\langle B \rangle + \langle Q_d \rangle + \sigma_{RO}^2) / N_{23}}_{\text{Poisson noise within } r_2 < r < r_3}$$

$$\underbrace{N_1 = \pi r_1^2}_{\text{Star}}, \quad \underbrace{N_{23} = \pi r_3^2 - \pi r_2^2}_{\text{Sky}}$$

- How do we choose r_1, r_2, r_3 ?
 - Signal increases with N_1
 - Noise increases with N_1 and decreases with N_{23}



Signal-to-Noise

$$SNR = \frac{\overbrace{F_N}^{\text{Signal}}}{\sqrt{\underbrace{F_N}_{\text{Signal Noise}} + \underbrace{N_1(\langle B \rangle + \langle Q \rangle + \sigma_{RO}^2)}_{\text{Sky Dark \& RON in star aperture}} + \underbrace{N_1(\langle B \rangle + \langle Q \rangle + \sigma_{RO}^2) / N_{23}}_{\text{Sky Dark \& RON in sky aperture}}}}$$

- How do we choose r_1, r_2, r_3 ?
 - Signal increases with N_1
 - Noise increases with N_1 and decreases with N_{23}

An Example

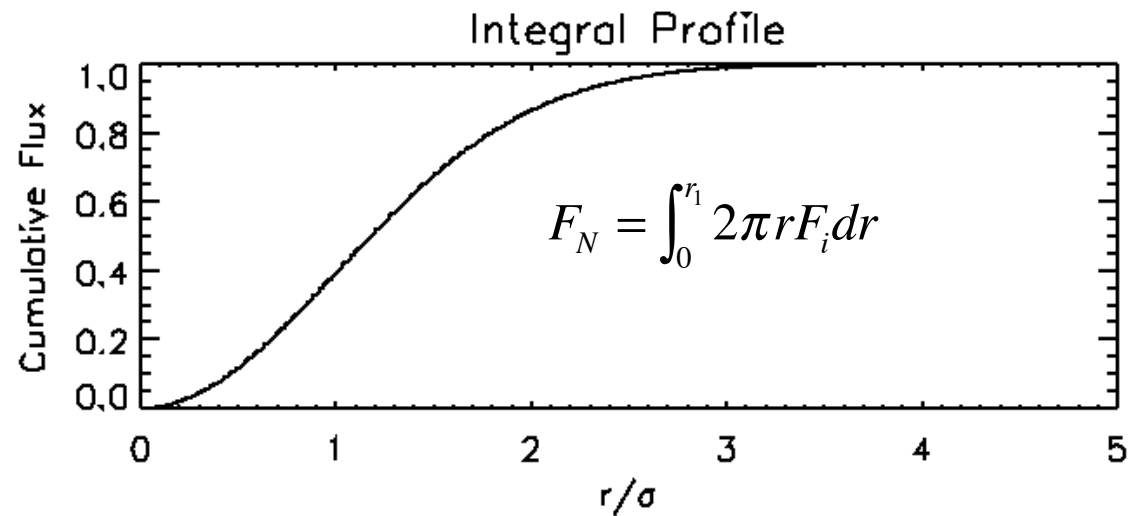
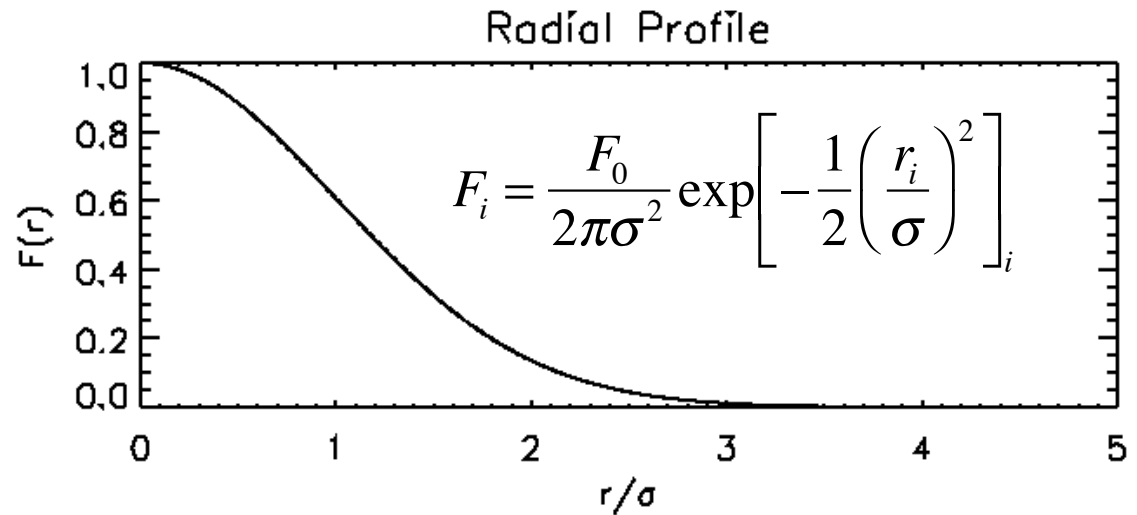
- Suppose the stellar signal has a 2-d Gaussian shape

$$F_i = \frac{F_0}{2\pi\sigma^2} \exp\left[-\frac{1}{2}\left(\frac{r_i}{\sigma}\right)^2\right], \quad r_i^2 = (x - x_0)^2 + (y - y_0)^2$$

$$F_N = \int_0^{r_1} 2\pi r F_i dr$$

- This tells us how F_N changes with aperture radius

Star Profile & Integral



SNR vs. r_1

- $F_0 = 100 e^-$
- $B_i = 100 e^-$
- $I_i = 0 e^-$
- $\sigma_{RO} = 10 e^- rms$
- $N_{23} \gg N_1$

