A CCD Noise Model

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2009/9/29

1 Signal

The CCD in the spectrometer employs the photovoltaic effect to measure the intensity of light, I_ν, at frequency ν. The number of photoelectrons in a given pixel, N_{pe}, per exposure depends on the integration time, Δt, the solid subtended by a pixel, $\Delta\Omega$, and the spectral band-pass, $\Delta\nu$, covered by that pixel,

$$N_{pe} = \eta I_\nu \cos\theta \Delta\nu \Delta t \Delta\Omega / h\nu \ , \tag{1}$$

where θ is the angle between pixel normal and the incident beam, η is the quantum efficiency (η = 1 for a perfect pixel), and h is Planck's constant. Each pixel also generates a "dark current", i_d, even when there is no illumination, so that the total charge is $N = N_{pe} + N_d$, where $N_d = i_d \Delta t$.

During readout of the CCD pixel the accumulated electric charge is deposited on a capacitor of capacitance C, thereby generating a voltage

$$V_{pe} = Ne/C \ , \tag{2}$$

where e is the charge of the electron. Typical values of C are a few pF; one electron (1.60×10^{-19} C) on 1 pF generates a signal of 160 nV.

The signal generated by the spectrometer is a number returned by a digital circuit that converts voltage to a 12-bit number (0-4095): this number is directly proportional to the voltage. By convention, the signal from the analog to digital converter (ADC) is measured in *ADU*s (analog to digital units). The voltage measuring circuit has a constant of proportionality, g, with units of *ADU* per volt. Thus, the number that ends up in your data file is

$$ADU = g\, Ne/C + ADU_0 \ . \tag{3}$$

The quantity ADU_0 is an offset or bias—a non-zero count that is returned even when the number of photoelectrons is zero. Note that the combined quantity ge/C has units of *ADU* per electron. For convenience, this quantity is often known simply as the gain.

We can use our knowledge of Poisson statistics applied to counting photoelectrons to deduce the gain factor in Eq. (3). First, notice that Eq. (3) implies that the signal in *ADU* depends on the number of photoelectrons and the bias value, i.e.,

$$ADU = ADU\left(N, ADU_0\right) . \tag{4}$$

To find the error in the measured signal we can apply the fundamental formula for error propagation, which states that the variance in some quantity f, which is a function of u, v, w, … is

$$\sigma_f^2 = \left(\frac{\partial f}{\partial u}\right)^2 \sigma_u^2 + \left(\frac{\partial f}{\partial v}\right)^2 \sigma_v^2 + \left(\frac{\partial f}{\partial w}\right)^2 \sigma_w^2 + \dots \ , \qquad (5)$$

where we have assumed that u, v, w, … are independent quantities with zero covariance.

2 Error propagation

Two types of noise contribute to the standard deviation of *ADU* measured, or σ_{ADU}. By applying the law of error propagation, Eq. (5), to Eq. (3) we find

$$\sigma_{ADU}^2 = \left(\frac{\partial ADU}{\partial N}\right)^2 \sigma_N^2 + \left(\frac{\partial ADU}{\partial ADU_0}\right)^2 \sigma_0^2 \ , \qquad (6)$$

again assuming zero covariance between these two noise sources. The first partial derivative is just the gain, ge/C, while the second is unity. The variance σ^2_N in the first term is associated with Poisson noise, for which we know that $\sigma^2_N = N$. In general there is some noise associated with each measurement, σ_0, which is known as the read noise. Thus, Eq. (6) simplifies to

$$\sigma_{ADU}^2 = \left(\frac{ge}{C}\right)^2 N + \sigma_0^2 \ . \qquad (7)$$

By substituting $geN/C = ADU - ADU_0$ from Eq. (3) we have

$$\sigma_{ADU}^2 = \frac{ge}{C}\left(ADU - ADU_0\right) + \sigma_0^2 \ . \qquad (8)$$

Thus, a plot of the variance, σ^2_{ADU}, versus the bias subtracted signal, $ADU - ADU_0$, should be a straight line with slope equal to ge/C and intercept of read noise squared.

3 A few notes

The bias is measured by taking short exposures. It is best to perform this experiment to measure the gain and read noise with a bright source and short exposures (10 ms) so that dark current is negligible. In this case, the quantity ADU_0 is measured by turning off your light source and repeating the experiment with the same exposure time.

We have assumed that bias subtraction is perfect and that ADU_0 is known with perfect precision. Is this a correct assumption?