Lab #3:
Astrometry from CCD Images

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Your report is due on October 27, 2009 at 5:59 PM PDT. Please—no extensions!

1 Overview

In this lab we will use CCD images to measure the positions of stars relative to the celestial coordinate system. We will use data from a 24-inch robotic telescope located on Kitt Peak, AZ. The purpose of this activity is to learn astronomical skills that we will apply in Lab 4, where we will track the position of some minor planets (asteroids) over the course of three to four weeks, and thereby determine the orbital elements of these bodies. For this lab we will use some data that is already in hand that was acquired in late September. At the end of this lab you will request observations that will comprise the final epoch of observations for the orbit determination.

Sections 2-5 of this document provide a step-by-step description of: raw astronomical CCD images; CCD calibration and correction; measurement of star positions and comparison to standard star catalogs; and finally, measurement of the position of an asteroid.

1.1 Key steps

The key steps are:

1) Read FITS files from the CCD on the 24-inch telescope and display the data using IDL (§3.1);
2) Apply systematic corrections including bias and flat field (§3.3);
3) Compare your asteroid field image with the digital sky survey and confirm that field is correct (§4);
4) Measure the positions of the stars in your CCD image (§4.1);
5) Cross-correlate the stars in your image with those in the USNO B-1 catalog (4.2);
6) Compute the standard coordinates for the USNO B-1 stars and match the two lists (§4.3);
7) Perform a least squares fit to find the "plate constants"(§5);
8) Examine the residuals between the measurements and the fit;
9) Find the asteroid;
10) Compute the position (and associated errors) of the asteroid;
11) Schedule the next epoch observations of your target.

1.2 Schedule

This is a three-week lab—10/6, 10/13, & 10/20. Your lab report is due on 10/27. For "show-and-tell" on 10/13 you should have progressed at least to step 6, and on 10/20 you should be prepared to discuss the residuals in your least squares fit and the error in the position of the asteroid.

2 The Observatory

We will be using the Super-LOTIS (Livermore Optical Transient Imaging System[1]). Super-LOTIS is a 24-inch (0.6-m) robotic telescope dedicated to the search for optical counterparts of gamma-ray bursts (GRB). The telescope is housed in a roll-off-roof facility at the Steward Observatory Kitt Peak site near Tucson, AZ. The experiment is a collaboration between Steward Observatory, Lawrence Livermore National Laboratory, NASA GSFC, Clemson University, and UC Berkeley. In addition to observing GRBs, Super-LOTIS performs nightly observations of science targets in a queue mode. This is the mode that we will be using.

Figure 1: A webcam picture from inside the dome of the 24-inch Super-LOTIS telescope on Kitt Peak.

The telescope is equipped with a 2048×2048 pixel CCD. The resultant field of view is approximately 17×17 arc minutes[2]. Details of the facility are listed in Table 1.

Table 1: Nominal observatory and instrument properties

Site	Kitt Peak, AZ
Geographic location	31:58:48 +111:36:00
Primary mirror	24-inch/0.6-m (diameter)
F/ratio-focal length	F/9 – 5570 mm
Camera	Spectral Inst[2]. 800 Series CCD
Detector	E2V TE Cooled CCD
CCD format	2048×2048, 13.5 μm pixels
Pixel scale	0.5 arc seconds/pixel
Field-of-view	17 arc minutes (square)
Optical Filters	V, R, I, and H-alpha (656 nm)
Limiting Magnitude	$R \sim 17.5/18.5$ mag. (10/60 s)

[1] http://slotis.kpno.noao.edu
[2] http://www.specinst.com/

3 Images

CCD images are saved on disc in a standard astronomical format, known as FITS files. FITS files contain the image data, stored in binary format, together with some information that records the circumstances under which the data were obtained. Scan the observing log for the night you are interested in, and select a relevant exposure that contains observations of an asteroid. A typical log is shown in the Appendix 6.

3.1 FITS headers

Although the image data in FITS files are in binary format, each file also has an ASCII preamble that can be conveniently inspected at the Unix command line by typing, for example:

```
ugastro% fold flat006.fits | more
SIMPLE  =                    T / file does conform to FITS standard
BITPIX  =                   16 / number of bits per data pixel
NAXIS   =                    2 / number of data axes
NAXIS1  =                 2200 / length of data axis 1
NAXIS2  =                 2200 / length of data axis 2
EXTEND  =                    T / FITS dataset may contain extensions
BZERO   =                32768 / offset data range to that of unsigned short
BSCALE  =                    1 / default scaling factor
DATE    = '2008-09-26T01:39:25' / file creation date (YYYY-MM-DDThh:mm:ss UT)
INSTRUME= 'SI Model 802 SN 157'
DATE-OBS= '2008-09-26T01:39:12.445' / exposure start time
EXPTIME =                3.000 / [s] exposure time in seconds
CCDTEMP =                238.3 / [K] CCD temperature
TECTEMP =                301.5 / [K] backplate temperature
VACUUM  =                 0.01 / [torr] vacuum
BIASSEC = '[2105:2195,5:2195]' / bias section
TRIMSEC = '[51:2098,1:2048]'   / trim section
DATASEC = '[51:2098,1:2048]'   / actual data pixels in the raw frame
CCDSEC  = '[1:2048,1:2048]'    / section of the detector used
ORIGSEC = '[1:2048,1:2048]'    / original size of the detector
GAIN    =                 3.93 / gain, electrons per ADU
RDNOISE =                11.59 / 800 kHz read noise, electrons
OBSERVAT= 'Steward KP, Super-LOTIS' / observatory
TELESCOP= 'Super-LOTIS 60-cm'  / telescope
OBSERVER= 'Peter A. Milne, U of A' / PI
TCS_RA  = '182602.92'
TCS_DEC = '+264707.1'
RA      = '18:26:02.92'
DEC     = '+26:47:07.1'
UT      = '01:39:33.3'         / TCS UT
AZ      = '-161.3  '
EL      = '84.8    '
TELFOCUS= '130330  '
LST     = '18:33:54'
HA      = '+00:07:32'
JD      = '2454735.5'
TS      = '1222393165'
AIRMASS = '1.00    '
EQUINOX = '2000.000'
RH      = '43.6    '           / Relative Humidity
WINDDIR = '273.8   '           / Wind Direction
WINDSPD = '2.8     '           / Wind Speed
ROOFSTAT= '0.0000  '           / Open=0.0...Closed=1.0
FILTER  = 'V       '
INSTFILT= '3,6     '
OBJECT  = 'Flat_V  '
END
```

The header includes useful information about the size of the image (NAXIS1 & NAXIS2), the date (DATE-OBS), the exposure time (EXPTIME), where the telescope was pointing (RA & DEC), and the optical filter in the light path (`FILTER`). The header contains "keywords," e.g., `FILTER` that have values on the right hand side of the equals sign.

The image data and the keywords are read into IDL using the function `READFITS()`, for example to read the file `image101.fits` into the variable `x` type

```
IDL> x = readfits('image101.fits',hdr)
% READFITS: Now reading 2200 by 2200 array
IDL> help,x
X               UINT      = Array[2200, 2200]
IDL> print, mean(x)
      712.742
```

(What type of variable is a `UINT`?) Scrolling through the ASCII FITS header in a terminal is fine for browsing, but if you want to use information in the FITS header in your IDL program, you will need to read the keywords. This is accomplished with the IDL function `SXPAR`, e.g., to find the object name

```
IDL> x = readfits('image101.fits',hdr)
% READFITS: Now reading 2200 by 2200 array
IDL> print, sxpar(hdr,'OBJECT')
Cerberus
```

3.2 Data regions and the CCD bias

The Super-LOTIS FITS files contain images that have dimensions 2200×2200, even though there are only 2048×2048 pixels in the CCD array. Inspection of the FITS header shows the following information:

```
BIASSEC = '[2105:2195,5:2195]' / bias section
TRIMSEC = '[51:2098,1:2048]'   / trim section
DATASEC = '[51:2098,1:2048]'   / actual data pixels in the raw frame
CCDSEC  = '[1:2048,1:2048]'    / section of the detector used
ORIGSEC = '[1:2048,1:2048]'    / original size of the detector
```

The actual data are located between pixels `[51:2098,1:2048]` (note that the `FITS` convention is that the first pixel is labeled 1 not 0!). The data values in the pixel range `[2105:2195,5:2195]` are measurements of the bias level. The signal in this region should be averaged and subtracted from each exposure to remove the CCD bias.

3.3 Bias, darks, & flats

At the beginning of each night various calibration frames, including bias, darks, and flats are acquired. The bias frames are zero second exposures and provide an additional measure of the digital offset when the signal is zero. For dark exposures the shutter does not open; these images can be used to measure the dark current. Flats are twilight sky images—both evening and morning twilight flats are available.

Dark frames are also listed at the start of the log file. Typically, 60 s darks are obtained. The Super-LOTIS CCD is cooled by a thermoelectric cooler with nominal operating temperature of about 240 K, which ensures that the dark current is negligible. Be sure to check the keyword:

```
CCDTEMP =                238.3 / [K] CCD temperature
```

to confirm that the cooler is turned on. A histogram of dark current is shown in Figure 2 (what does this plot say about the accuracy of the bias subtraction?). In a 60 s exposure the dark amounts to only 3.2 ADU, which is negligible compared to the sky brightness. When using short exposures (a few minutes or less) and a cooled CCD, dark subtraction can probably be neglected.

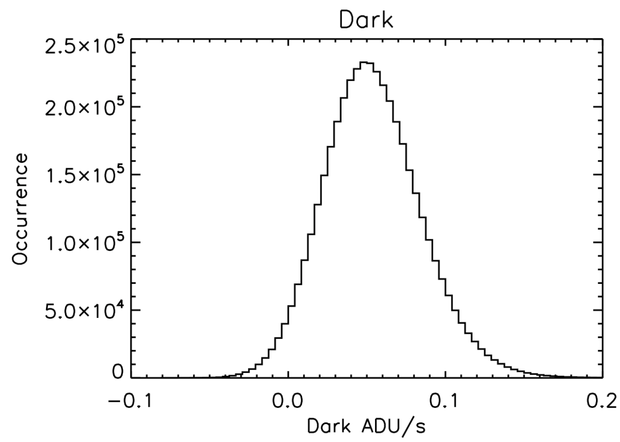


Figure 2: A dark histogram derived from the average of four bias-subtracted 60 s dark exposures.

The twilight sky provides a convenient source of spatially uniform illumination to measure the pixel-to-pixel response of the camera. Twilight images acquired for the purpose of making a flat field are listed as "flats" at the head of the log file. The median brightness in the *V*-band frames from 2008-09-26 UT designated as flats are shown in Figure 3. There are two sequences in this example: the short exposures (3–8 s) were taken in evening twilight and have a median level of about 6000 ADU, and a second sequence of frames that were acquired the following morning, which have only a few hundred counts. In this example, only the evening twilight frames are useful. The telescope tracking is turned off for flats, so stars appear to move from one image to the next (see Figure 4).

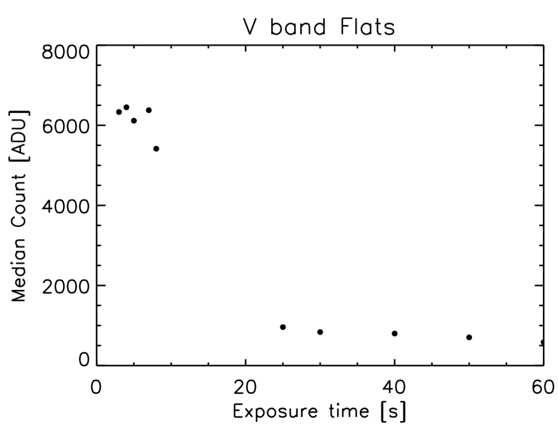


Figure 3: Median brightness for the bias-subtracted *V*-band "flat" images taken in twilight (2008/9/26 UT) at Super-LOTIS data. These exposures were taken during twilight. The first image in the sequence is a 3-s exposure, which was taken at 2008-09-26 01:39:25 UT. At that time the sun was about 5° below the horizon, i.e., just before the start of civil twilight. The long exposures were take the following morning at 12:41 UT when the sun was 8° below the horizon. These images have insufficient counts to make a high signal-to-noise flat.

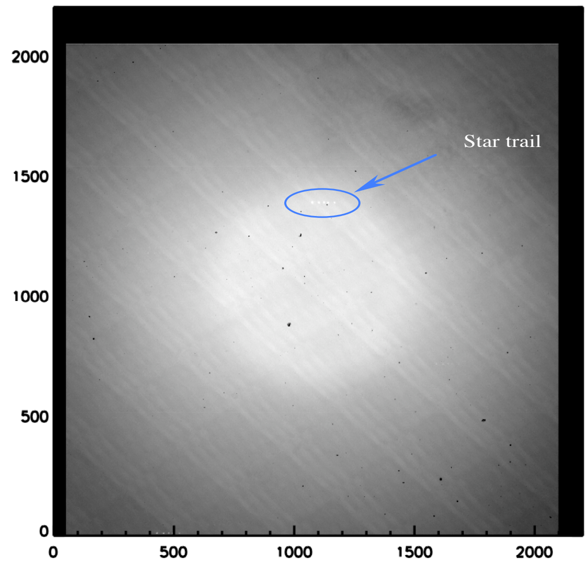


Figure 4: An average of five evening twilight sky *V*-band images from 2008-09-26 UT (flat006.fits – flat010.fits). The images have been bias subtracted using the bias region on the right. The grey scale runs from 5000 ADU (black) to 7500 ADU (white). Note the overall bulls-eye illumination pattern, which corresponds to 30% variation in response from center to edge. Note also the small-scale diagonal pattern. In addition there are some local low spots where the response is poor. Near pixel (1120 , 1400) the image of a star trail can be seen.

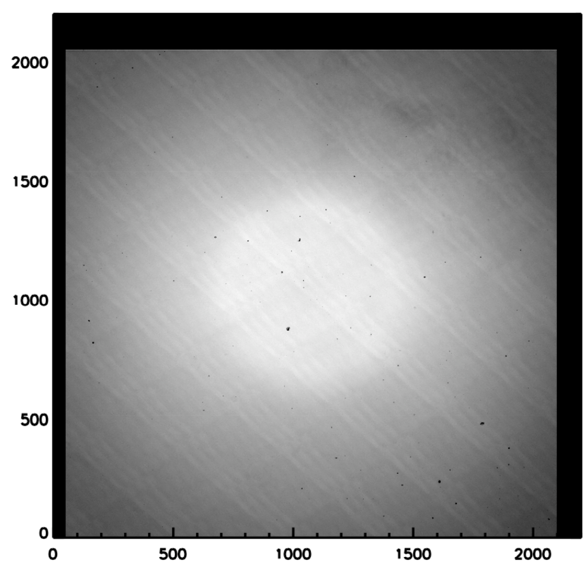


Figure 5: A *V*-band flat constructed from the same five frames shown in Figure 4, but combined using a median rather than an average. Each frame was scaled to a median value of 1 before median combining the stack. Note that the star trail visible in Figure 4 has disappeared. The greyscale runs from 0.8 (black) to 1.2 (white).

4 A star field

Now that we are ready with a flat field we can take a look at one of the target fields. In this example we choose a 10 s *R*-band image, subtract the bias, and divide by the appropriate flat:

```
IDL> x = readfits(‘image151.fits’,hdr)
IDL> biasval = median(x[2104:2194,4:2194]) ; bias subtract
IDL> x = x - biasval
IDL> flat = readfits('Flat_R-med-sky-flat.fits')
IDL> x = x/flat
IDL> xr = rotate(x,5)
```

According to Table 1 the limiting magnitude in this filter for a 10 s exposure should be about 17.5 mag. in *R* band. Inspection of this image (Figure 6) reveals about 200 stars. Note that the raw images have to be flipped left-right using the rotate command so that images appear in conventional orientation with north at the top and east to the left.

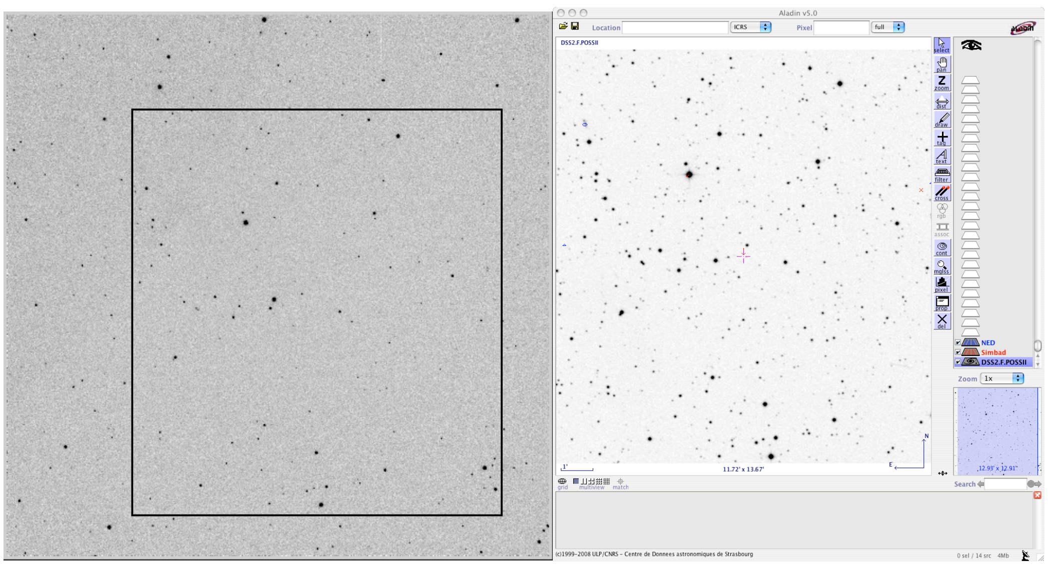


Figure 6: *Left:* A 10-s *R*-band image (image151.fits) from 2008-09-26 UT. The grey scale has been flipped so that stars show up as black dots. *Right:* The digital sky survey image for a 11.7 × 13.7 arc minute square field at 05:06:09.9 +25:45:35 2000.0 from ALADIN[3]. The inset square on the left shows the corresponding region. This comparison suggests that the field center is closer to 05:06:12.6 +25:46:16 2000.0.

4.1 Star positions

Once we have confirmed that the image corresponds approximately to the correct field, the next step is to identify and measure the *x*- and *y*-positions of the stars in the image. The location of stars are best measured using their centroids:

$$\langle x \rangle = \sum_i x_i I_i \Big/ \sum_i I_i\,, \quad \langle y \rangle = \sum_i y_i I_i \Big/ \sum_i I_i\,, \tag{1}$$

[3] http://aladin.u-strasbg.fr/aladin.gml

where the summation, i, runs only over a region in the vicinity of the star. Note that the denominator, ΣI_i, is a measure of the brightness of the star—this is useful information that should be saved along with the centroids.

4.2 Star catalogs

We can now compare the observed x- and y-positions on our CCD image with the predicted positions of stars taken from a catalog, such as the US Naval Observatory USNO-B1.0 Catalog[4]. The USNO-B1.0 catalog lists positions, proper motions, and magnitudes in various optical pass-bands for 1,042,618,261 objects over the entire sky. The data are from scans of photographic plates from the Palomar 48-inch Schmidt telescope taken for various sky surveys during the last 50 years. The USNO-B1.0 catalog provides all-sky coverage down about $V = 21$ mag., and astrometric accuracy of 0.2 arc second at J2000, and should be adequate for our purposes.

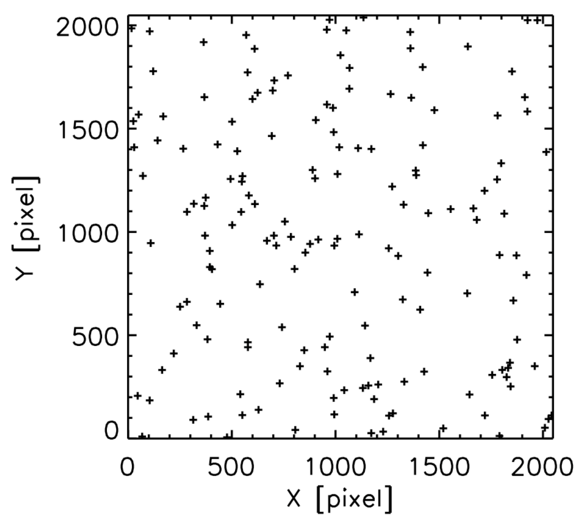


Figure 7: The positions of stars in our Super-LOTIS CCD image located from Figure 6.

You can interrogate the USNO-B1.0 catalog from within IDL via the Vizier[5] astronomical catalog database using the IDL function `queryvizier`[6] as follows:

```
rast   = sxpar(hdr,'RA') ; get the position from the FITS header
decst  = sxpar(hdr,'DEC')
; Parse the strings into decimal degrees---don't forget the
; factor of x15 to convert from HRS to DEG
radeg  = 15*ten(strmid(rast,0,2),strmid(rast,3,2),strmid(rast,6,5))
decdeg = ten(strmid(decst,0,3), strmid(decst,4,2),strmid(decst,7,4))
fov   =[17.0, 17.0] ; Field width (arcmin) of box to search
usno = QUERYVIZIER('USNO-B1',[radeg,decdeg],fov) ; Download USNO stars
```

The IDL function `queryvizier` returns an object known as a structure. To find out what components comprise the structure type:

[4] http://tdc-www.harvard.edu/software/catalogs/ub1.html
[5] http://vizier.hia.nrc.ca/viz-bin/VizieR
[6] http://idlastro.gsfc.nasa.gov/ftp/pro/sockets/queryvizier.pro

```
IDL> help,usno,/struct
** Structure <229ee84>, 14 tags, length=72, data length=72, refs=1:
   USNO_B1_0       STRING    '1157-0004938'
   RAJ2000         DOUBLE           4.9504640
   DEJ2000         DOUBLE           25.761353
   E_RAJ2000       LONG               230
   E_DEJ2000       LONG               265
   EPOCH           FLOAT            1969.30
   PMRA            LONG                 0
   PMDE            LONG                 0
   NDET            LONG                 3
   B1MAG           FLOAT            20.6200
   R1MAG           FLOAT            19.4000
   B2MAG           FLOAT                NaN
   R2MAG           FLOAT            19.7300
   IMAG            FLOAT                NaN
```

Refer to an element of a structure using a period; thus, to reference the right ascension use `usno.raj2000`. So to examine the stars that you've found you can plot them:

```
w = where(usno.r2mag le 14.) ; pick stars brighter than 14
plot,usno[w].raj2000,usno[w].dej2000,ps=1,xtitle = 'RA [deg]',$
ytit ='DEC [deg]',/iso,/ynoz,title = 'USNO B-1 stars',charsiz=2,$
xr = [max(usno[w].raj2000),min(usno[w].raj2000)]
```

Note that by convention right ascension increases to the left. The resultant plot for this example is shown in Figure 8. Comparison of Figure 7 and Figure 8 does not immediately give the impression that we are looking at the same star field. To make sure you have the right stars you can interrogate the USNO B-1 catalog directly from ALADIN, using the open/surveys menu choice. Clicking on a star in the ALADIN window will show you the catalog entry.

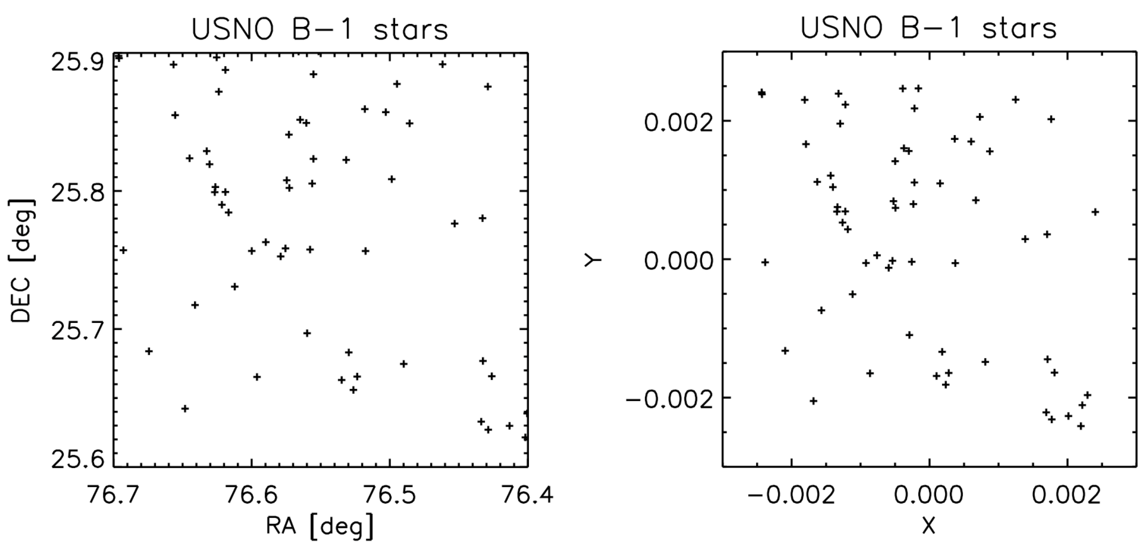


Figure 8: ***Left*****: The catalog positions of 62 USNO B-1 stars within a 17 × 17 arc minute square field centered at 05:06:09.9 +25:45:35 2000.0 with magnitude brighter than 15.0.** ***Right:*** **the same data plotted in standard coordinates (see §4.3)—the projection from celestial coordinates on the unit sphere tangent plane.**

4.3 Matching stars & standard coordinates

We now have the positions of stars on the CCD in pixel *x* and *y* and in right ascension and declination from the USNO B-1 catalog. The USNO B-1 coordinates are on the celestial sphere, whereas the stars in our CCD image are measured on a plane, which is a projection of the celestial sphere. The first step in the comparison is to project the USNO B-1 coordinates from the celestial sphere to the plane represented by the CCD so that we can make a one-to-one comparison of these quantities.

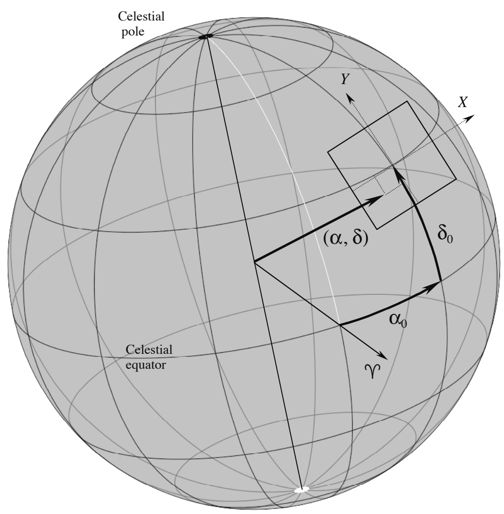


Figure 9: Projection from celestial coordinates of a point (α, δ) on the sky in a field centered at (α_0, δ_0) onto the tangent plane of a CCD with a position (*X*, *Y*). The *X*-axis is aligned along the small declination circle and the *Y*-axis is aligned along the great hour circle, and points towards the celestial pole. The symbol ♈ indicates the direction of the vernal equinox, which defines the origin of right ascension.

The transformation can be obtained by considering the geometry in Figure 9 (see the detailed derivation in §7). The projected coordinates (*X*, *Y*) of a position on the celestial sphere (α, δ) in the vicinity of a field centered at (α_0, δ_0) are

$$\begin{aligned} X &= -\frac{\cos\delta\sin(\alpha-\alpha_0)}{\cos\delta_0\cos\delta\cos(\alpha-\alpha_0)+\sin\delta\sin\delta_0} \\ Y &= -\frac{\sin\delta_0\cos\delta\cos(\alpha-\alpha_0)-\cos\delta_0\sin\delta}{\cos\delta_0\cos\delta\cos(\alpha-\alpha_0)+\sin\delta\sin\delta_0}, \end{aligned} \tag{2}$$

where the celestial sphere is taken to be the unit radius.

Conversion from (X, Y) coordinates to pixel coordinates (x, y), is straightforward. If f is the focal length of the camera and p is the pixel size, then for an ideal camera

$$\begin{aligned} x &= f\left(X/p\right) + x_0 \\ y &= f\left(Y/p\right) + y_0 \end{aligned},$$

where, by ideal we mean that the focal length of the imaging system is constant over the field, there is no anamorphic magnification ($f_x = f_y$), or equivalently the pixels are square ($p_x = p_y$), and that the CCD is oriented so that the X-axis lies along the declination small circle. In general none of these conditions are true. Optical systems have variable magnification and distortion, pixels may be parallelograms, and the CCD is subject to an unknown rotation with respect to celestial coordinates. However, for a first guess let's adopt the nominal values from Table 1: $f = 5570$ mm and $p = 0.0135$ mm. The results of this comparison are shown in Figure 10.

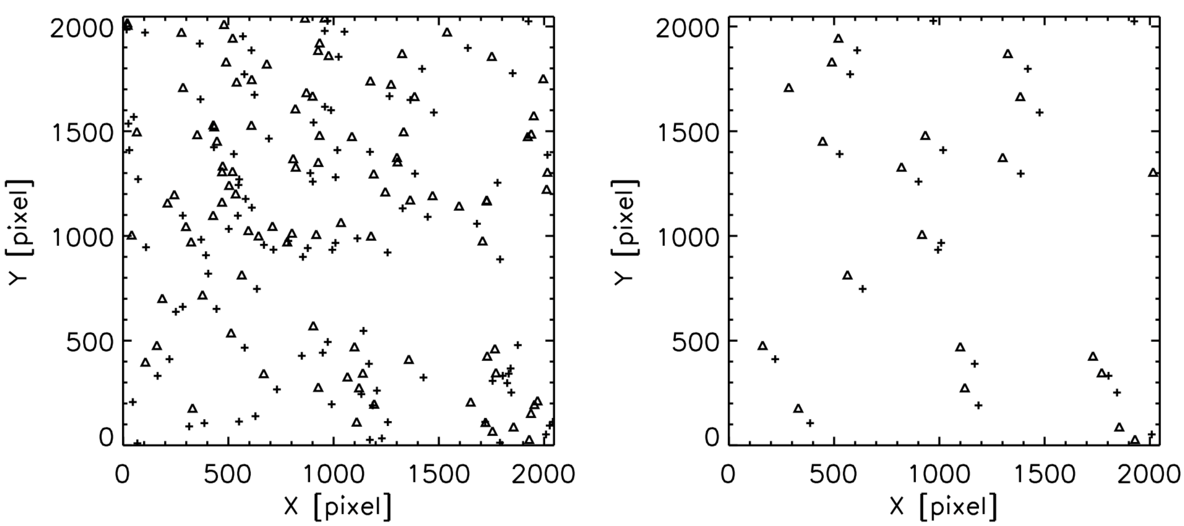


Figure 10: ***Left:*** **Comparison of the measured pixel positions (cross) of stars and converted standard positions from the USNO B-1 catalog (triangle) using $f = 5570$ mm, $p = 0.0135$ mm, and $x_0 = y_0 = 1024$. The situation is obviously complicated.** ***Right:*** **Plotting the brightest twenty stars only reveals that the two fields differ by a small offset.**

Evidently, if we plot all the stars (left) then it is difficult to figure out how the star patterns match up; however, if we choose only the twenty brightest stars from each list it is clear that that we see the same pattern with an offset. Applying an offset of (85, -70) pixels to the converted standard coordinates produces Figure 11, which shows that the nominal values of the focal length and the pixel size give a fairly good approximation to the expected position of the stars computed from USNO B-1 catalog coordinates. Inspection of the stars at the bottom edge of the image suggests that the two frames are rotated with respect to one another.

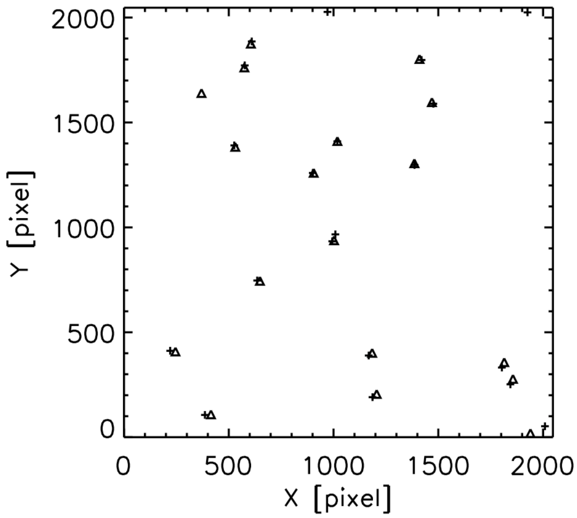


Figure 11: Same as Figure 10 (right), except that the converted standard coordinates have been shifted by (85, -70) pixels. It is now evident that the two star fields match up. The asteroid is the object at the center of the frame with no corresponding star.

5 Least squares

The standard coordinates are defined for the tangent plane to the unit sphere. As we have seen in § 4.3 the positions on the CCD depend on the focal length and the pixel size. In general the CCD is not perfectly aligned so there is a rotation between the (x, y) CCD-based coordinate system and the (X, Y) standard coordinates. Taking this into account

$$\begin{aligned} X &= \frac{p}{f}\left(x\cos\theta - y\sin\theta - x_0\right) \\ Y &= \frac{p}{f}\left(x\sin\theta + y\cos\theta - y_0\right) \end{aligned},$$

where θ is the rotation between the two frames. More generally let us suppose that we can write

$$\begin{aligned} X &= a_{11}x + a_{12}y + X_0 \\ Y &= a_{21}x + a_{22}y + Y_0 \end{aligned}.$$

Thus, in the linear approximation there are six unknown "plate constants", which we can solve for using the method of linear least squares. The constants a_{ij} refer to the scale, shear and orientation of the image, and X_0 and Y_0 are pointing offsets.

To find the least-squares solution we write down the equations of condition describing the relation between the independent variables (X_i, Y_i) and our N position measurements from the CCD image, (x_i, y_i), as

$$\begin{aligned} X_1 &= a_{11}x_1 + a_{12}y_1 + X_0 \\ X_2 &= a_{11}x_2 + a_{12}y_2 + X_0 \\ \cdots &= \cdots \\ X_N &= a_{11}x_N + a_{12}y_N + X_0, \end{aligned} \tag{3}$$

and

$$\begin{aligned} Y_1 &= a_{21}x_1 + a_{22}y_1 + Y_0 \\ Y_2 &= a_{21}x_2 + a_{22}y_2 + Y_0 \\ \cdots &= \cdots \\ Y_N &= a_{21}x_N + a_{22}y_N + Y_0. \end{aligned} \tag{4}$$

Or in compact matrix form for Eq. (3) for X can be represented as

$$\mathbf{X} = \mathbf{Ab},$$

where

$$\mathbf{X} = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots \\ x_N & y_N & 1 \end{pmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} a_{11} & a_{12} & X_0 \end{pmatrix},$$

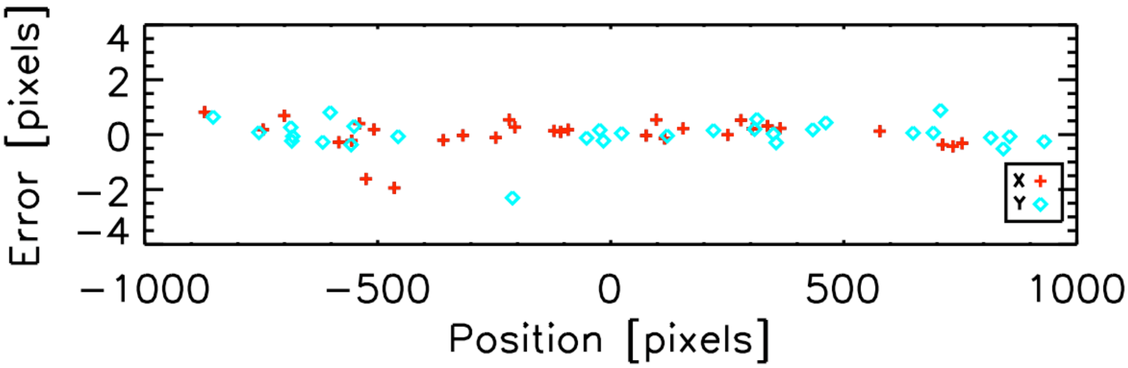
with a similar equation for Y.

This is not a problem that we can solve with built in IDL functions, such as `linfit` or `poly_fit`. However, as we will see in class, we can solve for **b** from by multiplying each side by the transpose of **A**, $\mathbf{A}^T$, followed by the inverse matrix $(\mathbf{A}^T\mathbf{A})^{-1}$, i.e.,

$$\mathbf{b} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{X}. \tag{5}$$

A similar expression is used for Y.

The results of applying Eq. (5) to our data set are summarized in Figure 12. In the initial match most stars show very small residuals, suggesting that the match is reliable except perhaps for three stars that may be mismatched. When these are removed the overall quality of the fit is improved.



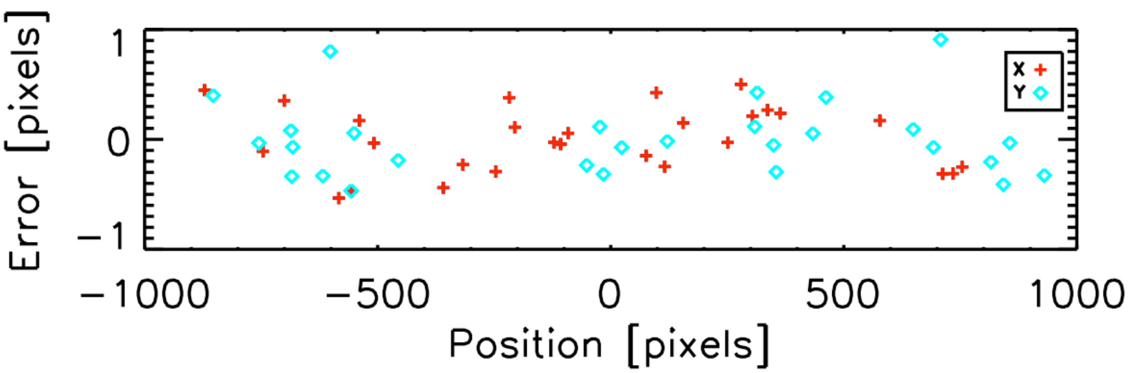


Figure 12: ***Top:*** **Errors in the standard coordinates in *X* and *Y* (converted to pixel units) using a linear least squares solution. In the top plot the 40 brightest stars are matched between the CCD and the USNO B catalog. Of these matches three have suspiciously large residuals. *Bottom:* If we remove matches with residuals larger than 1 pixel then the bottom plot results. The stars with poor matches (residuals greater than one pixel) may be false matches.**

The coefficients of the solution are shown in Table 2. An interesting quantity from the least squares solution is the focal lengths. Assuming that the pixel size is known with precision (p = 0.0135 mm) then the focal length is

$$f = 1\Big/\sqrt{\left|a_{11}a_{22} - a_{12}a_{21}\right|}\,.$$

Which yields f = 5620.6 mm, or about 0.9% shorter than the nominal value. Inspection of the matrix elements in Table 2 shows that it is very nearly skew symmetric, which means that pixels are square to better than a part in a thousand. The rotation of CCD columns from true north is about 1 degree.

Table 2: Least squares coefficients.

	X	Y
a_{ij}	1.7786E-04 -3.0828E-06	3.1131E-06 1.7791E-04
X_0, Y_0	-2.6056E-03	-2.3212E-03
rms error	0.0549 arc sec	0.0625 arc sec

The least squares solution allows us to compute the standard coordinates for any pixel in the CCD. The conversion from standard coordinates back to celestial coordinates is given by Eq. (10) (see §7).

The results of application of this method to the asteroid 10 Hygiea on UT 2008-9-26 over an interval of about 6 minutes is shown in Figure 13 and Table 3.

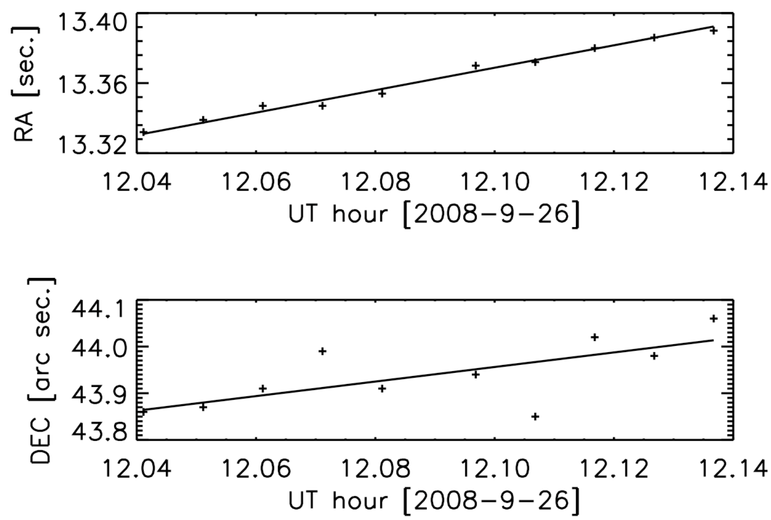


Figure 13: The right ascension and declination of 10 Hygiea listed in Table 3. The *y*-axis shows the right ascension seconds and declination seconds of arc. The motion of the asteroid over an interval of about six minutes is evident.

Table 3: Measurements of the position of 10 Hygiea (Equinox 2000) on UT 2008-9-26. Images 151-155 are *R*-band data and 156-160 are *I*-band. The rms values (in arc seconds) listed at the bottom of the table are relative to a straight line fit, which removes the linear motion of the target.

File number	RA J2000 (H M S)	DEC J2000 (D M S)	UT 2008-9-26 (H M S)
151	5 6 13.332	25 43 43.86	12 02 28
152	5 6 13.330	25 43 43.87	12 03 4
153	5 6 13.347	25 43 43.91	12 03 40
154	5 6 13.347	25 43 43.99	12 04 16
155	5 6 13.354	25 43 43.91	12 04 52
156	5 6 13.370	25 43 43.94	12 05 48
157	5 6 13.372	25 43 43.85	12 06 24
158	5 6 13.380	25 43 44.02	12 07 0
159	5 6 13.386	25 43 43.98	12 07 36
160	5 6 13.390	25 43 44.06	12 08 12
rms	0.032 arc sec	0.052 arc sec	

6 Appendix: The observing log

Each night an observing log is generated. Here is an extract from a typical example:

```
Image name     Target name           Filter  Exp(s)  Airmass
image120.fits  Cerberus              R       180.000 1.08
image121.fits  Asteroid_9_Metis      R       10.000  1.07
image122.fits  Asteroid_9_Metis      R       10.000  1.07
image123.fits  Asteroid_9_Metis      R       10.000  1.07
image124.fits  Asteroid_9_Metis      R       10.000  1.07
image125.fits  Asteroid_9_Metis      R       10.000  1.07
image126.fits  Asteroid_9_Metis      I       10.000  1.07
image127.fits  Asteroid_9_Metis      I       10.000  1.07
image128.fits  Asteroid_9_Metis      I       10.000  1.07
image129.fits  Asteroid_9_Metis      I       10.000  1.07
image130.fits  Asteroid_9_Metis      I       10.000  1.07
image131.fits  SGR0501_4516          R       120.000 1.06
```

These are self-explanatory, except perhaps for the airmass, which is $AM = 1/\cos(ZD)$, where ZD is the angular distance of the target from the zenith. In the flat-earth approximation AM is the atmospheric path length. A target directly overhead has $AM = 1$, i.e., the light from this object travels though one unit thickness of the earth's atmosphere. A target 60° from the zenith (elevation 30°) has $AM = 2$, and experiences a path length with twice as much atmospheric path length.

6.1 Calibration data

Bias frames (zero second exposures), dark fames (shutter closed), and twilight sky flats are listed at the start of the log file. For example:

```
bias1.fits     bias    R       0.000   1.01
bias2.fits     bias    R       0.000   1.01
bias3.fits     bias    R       0.000   1.01
bias4.fits     bias    R       0.000   1.01
bias5.fits     bias    R       0.000   1.01
dark1.fits     dark    R       10.000  1.02
dark10.fits    dark    R       60.000  1.02
dark2.fits     dark    R       10.000  1.02
dark3.fits     dark    R       10.000  1.02
dark4.fits     dark    R       10.000  1.02
dark5.fits     dark    R       10.000  1.02
dark6.fits     dark    R       60.000  1.02
dark7.fits     dark    R       60.000  1.02
dark8.fits     dark    R       60.000  1.02
dark9.fits     dark    R       60.000  1.02
flat011.fits   Flat_R  R       8.000   1.01
flat012.fits   Flat_R  R       9.000   1.01
flat013.fits   Flat_R  R       11.000  1.01
flat014.fits   Flat_R  R       16.000  1.01
flat015.fits   Flat_R  R       20.000  1.01
flat111.fits   Flat_R  R       8.000   1.01
flat112.fits   Flat_R  R       7.000   1.01
flat113.fits   Flat_R  R       5.000   1.01
flat114.fits   Flat_R  R       4.000   1.01
flat115.fits   Flat_R  R       3.000   1.01
```

Note that the files are not listed in chronologoical order. If you are looking for flats be sure that the dome is open, i.e., check the keyword:

```
ROOFSTAT= '0.0000  '           / Open=0.0...Closed=1.0
```

and make sure that the images have plenty of counts (~ 6000 ADU).

6.2 Automated flat generation

Here's an IDL script to generate a flat field from the twilight sky frames. Frames are selected to have counts above some threshold, bias subtracted, scaled to the median value, and then median combined. Using a median rather than an average eliminates any stars in the field.

```
; Automated sky flat generator : 10/1/2008
; first pick your Filter by uncommenting the keyword that identifies the
; frame as a V-, R-, or I-band flat

search_str = 'Flat_R  '
;search_str = 'Flat_V  '
;search_str = 'Flat_I  '

nx = 2200
ny = 2200
skythres = 3000.0 ; sky signal must be above this level to be included
; Range of filenames to consider
n0 =   1
n1 = 120
iflat = 0

; Loop over flat data and look for appropriate flats
for i = n0, n1 do begin
    fn = 'flat'+string(i,form='(i03)')+'.fits'
    print,'File ',fn
    x = readfits(fn,hdr,/silent)
    if n_elements(x) gt 1 then begin
       obj = sxpar(hdr,'OBJECT')
       if obj eq search_str and median(x) gt skythres then begin
          if iflat eq 0 then begin
             fns = fn
          endif else begin
             fns=[fns,fn]
         endelse
         iflat++
     endif
   endif
endfor
nf = n_elements(fns)
print,'There are ',nf,' flats brighter than ', skythres
flats = fltarr(nx,ny,nf)
for i=0,nf-1 do begin
   x = readfits(fns[i],hdr,/silent)
   biasval = median(x[2104:2194,4:2194]) ; bias subtract
   x -= biasval
   medval  = median(x[50:2097,0:2047]) ; scale to the median value
   x /= medval
   flats[*,*,i] = x  ; stuff the frame into a 3-d array
endfor
skyflat = median(flats,dim=3) ; combine the median image
writefits,strcompress(search_str,/rem)+'-med-sky-flat.fits',skyflat,hdr

end
```

7 Appendix: Tangent plane projection

Consider the Cartesian coordinate system (x, y, z) with the x-axis pointed towards some right ascension α_0, and the x-y plane coincident with the celestial equator. The geometry is illustrated in Figure 14.

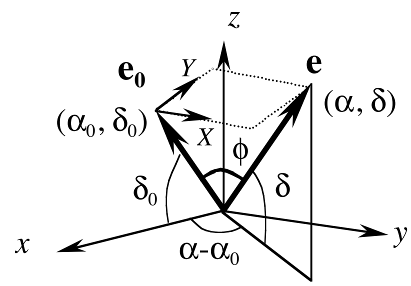


Figure 14: A coordinate system with the *x*-axis pointed towards some right ascension α_0, and the *x*-*y* plane coincident with the celestial equator. The angle between the directions (α, δ) and (α_0, δ_0) is ϕ. The tangent plane is normal to the unit vector $\mathbf{e}_0$. The tangent plane coordinates are (*X*, *Y*).

The components of the unit vector **e** pointing towards a star at (α, δ) are

$$\begin{aligned} x &= \cos\delta\cos(\alpha-\alpha_0) \\ y &= \cos\delta\sin(\alpha-\alpha_0)\ . \\ z &= \sin\delta \end{aligned} \tag{6}$$

The angle between the directions $\mathbf{e}(\alpha, \delta)$ and $\mathbf{e}_0(\alpha_0, \delta_0)$ is ϕ is given by computing the dot product

$$\begin{aligned} \cos\phi &= \mathbf{e}_0 \cdot \mathbf{e} \\ &= \cos\delta\cos(\alpha-\alpha_0)\cos\delta_0 + \sin\delta_0\sin\delta. \end{aligned} \tag{7}$$

Consider the plane normal to $\mathbf{e}_0$ with X oriented along the y-axis and Y rotated about y by an angle equal to δ_0 such that the unit vectors in this plane are

$$\hat{\mathbf{X}} = \begin{pmatrix}0 & 1 & 0\end{pmatrix}; \quad \hat{\mathbf{Y}} = \begin{pmatrix}\sin\delta_0 & 0 & -\cos\delta_0\end{pmatrix}.$$

A point in the tangent plane **p** is

$$\mathbf{p} = \mathbf{e}_0 - \left(X\hat{\mathbf{X}} + Y\hat{\mathbf{Y}}\right) = p\mathbf{e},$$

where $p = 1/\cos\phi = (1+X^2+Y^2)^{1/2}$.

When written in component form we have three simultaneous equations relating (α, δ) and (*X*, *Y*)

$$\begin{pmatrix} \cos\delta_0 - Y\sin\delta_0 \\ -X \\ \sin\delta_0 - Y\cos\delta_0 \end{pmatrix} = \begin{pmatrix} \dfrac{\cos\delta\cos(\alpha-\alpha_0)}{\cos\delta_0\cos\delta\cos(\alpha-\alpha_0)+\sin\delta\sin\delta_0} \\ \dfrac{\cos\delta\sin(\alpha-\alpha_0)}{\cos\delta_0\cos\delta\cos(\alpha-\alpha_0)+\sin\delta\sin\delta_0} \\ \dfrac{\sin\delta}{\cos\delta_0\cos\delta\cos(\alpha-\alpha_0)+\sin\delta\sin\delta_0} \end{pmatrix}, \tag{8}$$

which yield

$$\begin{aligned} X &= -\frac{\cos\delta\sin(\alpha-\alpha_0)}{\cos\delta_0\cos\delta\cos(\alpha-\alpha_0)+\sin\delta\sin\delta_0} \\ Y &= -\frac{\sin\delta_0\cos\delta\cos(\alpha-\alpha_0)-\cos\delta_0\sin\delta}{\cos\delta_0\cos\delta\cos(\alpha-\alpha_0)+\sin\delta\sin\delta_0} \end{aligned}, \tag{9}$$

or equivalently the inverse relation

$$\begin{aligned} \alpha &= \alpha_0 + \arctan\left[-\frac{X}{\cos\delta_0 - Y\sin\delta_0}\right] \\ \delta &= \arcsin\left[\frac{\sin\delta_0 + Y\cos\delta_0}{\left(1+X^2+Y^2\right)^{1/2}}\right] \end{aligned}. \tag{10}$$

8 Appendix: Web resources

Find what small bodies are observable at: http://ssd.jpl.nasa.gov/sbwobs.cgi

US Naval Observatory (home of the USNO B astrometric catalog): http://ad.usno.navy.mil/

ALADIN catalog and sky viewer: http://aladin.u-strasbg.fr/aladin.gml

9 References

Astronomy on the Personal Computer, Montenbruck & Pfleger, Springer (partial preview at http://books.google.com/books?id=WDjJIww337EC)

Modern Astrometry, Jean Kovalevsky, Springer (partial preview at http://books.google.com/books?id=s4azHlUeIYgC)