

THE HI LINE, THE GALAXY, and SUPERNOVA SHOCKS

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1. INTRODUCTION

Diffuse atomic gas permeates all corners of interstellar space. On the largest scales, its distribution reveals a lot about Galactic structure and dynamics. On smaller scales, its morphology is dominated by shocks from energetic stellar winds and supernovae. On still smaller scales, magnetic fields, pressure differences, and thermal instability shape the gas into thin filaments and sheets. On still smaller scales, the densities get large from self-gravitation; the gas turns molecular and stars form.

For this lab, we outline a several projects in HI that are suitable for our 4.5-m diameter Leuschner dish. They fall into two categories. One explores large-scale Galactic structure and dynamics by obtaining HI spectra along great circles, presenting the data as position-velocity images, and interpreting those maps as discussed below. The other maps specific objects by sampling a grid of spectra over a large area, presenting the data as grey-scale images of velocity-integrated line intensity (i.e., antenna temperature T_A) or as images in which image brightness indicates velocity-integrated T_A and color indicates velocity, and interpreting the data.

We'd like each group to pick a single project and carry it through to completion by the time the lab reports are due, which is the last day of instruction (May 1). All projects involve making maps and images. One of the features of this lab is to learn about image processing and using colors in images.

For the great-circle projects (§5 and §7), we gain large angular scale information in the sky by observing on great circles. Getting nice results requires data over the largest portions of the great circles you can get. For a full 360-degree circle, that's 180 pointings at 2° spacings. Mostly, though, we can't cover a full great circle because some of the circle lies too far south, so we do as much as we can.

2. SCHEDULE

The end of the semester is approaching like a freight train. Imagine yourself tied onto the tracks by a madman: you've gotta finish before it hits. Here's a schedule. If you don't keep up with this, you're in trouble!

1. Week 1, 02 April -9 April : Get the standard spectrum with the Leuschner dish and apply amplitude and velocity calibrations.

2. Week 2, 9 April -16 April : Pick a group project. Get the standard spectrum with the Leuschner dish. Figure out how you will get the data for your project. Get some data. Reduce the data.
3. Week 3, 16 April -23 April : Get the rest of your data. Figure out how you want to present the data as images. Make some first-try images.
4. Week 4, 23 April - 30 April: Finalize data. Make final images. Write report. Report is due the last day of class.

3. OBTAINING A CALIBRATED SPECTRUM

It's time to do astronomy! Point the dish to Galactic coordinates $(l, b) = (120^\circ, 0^\circ)$. The rotation matrix for conversion between equatorial and Galactic coordinates is in the coordinate-conversion handout. We'll take three separate measurements. Take plenty of samples—the more you take, the higher the signal-to-noise ratio.

1. Set the first local oscillator so that the HI line is roughly centered in the baseband spectrum. (The HI line frequency is 1420.4058 MHz). Because this is centered on the line, call this spectrum s_{online} .
2. Displace the first l.o. so that the line falls outside of the baseband spectrum. Because this is centered off the line, call this spectrum $s_{offline}$.
3. Repeat the offline spectrum with the cal on; call this one $s_{offline,calon}$.

Here is how you analyze this spectrum:

1. You can remove the instrumental bandpass, and thus get the shape of the line (but not the intensity), by taking the ratio

$$ratio = \frac{s_{online}}{s_{offline}} . \quad (1)$$

This is the *first* factor (i.e., the shape) in equation (13) in the spectral-line handout.

2. To get the line intensity in terms of the calibration noise source, multiply the ratio spectrum by T_{sys} —the second factor in equation (13). We obtain T_{sys} by combining the calon and caloff spectra, as per equation (15) in the handout:

$$T_{sys} = \frac{\sum s_{offline}}{\sum (s_{offline,calon} - s_{offline})} T_{cal} \quad (2)$$

where the numerator and denominator are summed over all of the channels, as in equation (15). We think T_{cal} is about 20 K; use this value for now¹. Then the final, intensity-calibrated spectrum is equation (16) in the handout, namely

$$final\ calibrated\ spectrum = ratio \times T_{sys} \quad (3)$$

3. Plot your final spectrum versus the sky frequency.
4. Plot your final spectrum versus the Doppler velocity². Remember that, by astronomical convention, positive velocity means motion away (remember the expansion of the Universe!), so

$$\frac{v}{c} = -\frac{\Delta f}{f_0}, \quad (4)$$

where c is the speed of light and Δf is the frequency offset from the line frequency f_0 .

5. Calculate the Doppler correction using the `ugdoppler` program (for documentation use `doc_library`). Correct the velocities to the Local Standard of Rest (LSR) and make a third plot.

Notes on ugdoppler:

1. You need the Julian day of the observation, which you get from IDL's `julday` function³. The Julian day is defined for *Greenwich*, whose time is 8 hours ahead of PST. That is, if the time in Berkeley is 5 pm (that's 17 hours) on March 17, then the time in Greenwich is 1 hour on March 18, and that's what you use as input to `julday`. You can get the current Julian day in IDL: `currentjulday = systime(/julian, /ut) .`
2. You need the observatory coordinates (north latitude and west longitude) in degrees; you could enter them with the pair of optional input parameters (`nlat`, `wlong`); the default values are Campbell Hall's values, so you need to input the correct values for Leuschner.

4. ON MAPPING AREAS: HOW TO SPATIALLY SAMPLE

All projects examine the angular dependence of the 21-cm line profile, so you need closely-spaced observations to make a map or an image. How closely should the observations be spaced?

¹You could attempt to calibrate T_{cal} by observing the Sun (or any source with known brightness temperature) and measuring the additional power associated with turning on the calibration load.

²Astronomers usually express velocities in km s^{-1} .

³If you have a vector of times for which you need the Julian day, use `timegen` instead.

The usual criterion is two samples per HPBW (Half-Power BeamWidth), because observing at more closely spaced angles just gives redundant information and takes extra time, while more widely spaced angles loses small-scale structural information. (This statement can be justified more exactly by Fourier transforming the structure of the sky in terms of angular frequency and then imposing the Nyquist criterion...). The HPBW is $\sim 4^\circ$; using a spacing of 2° is close to optimum, and it’s being a round number is nice.

For the mapping projects (not the great-circle ones), we need to cover areas of 1000 to 2000 square degrees. Consider the Orion-Eridanus Superbubble (§8 below), which wants to map the the region covering roughly ($\ell = 160^\circ \rightarrow 220^\circ, b = -70^\circ \rightarrow -10^\circ$). You might think you’d set up a regular grid in (ℓ, b) with $(\Delta\ell, \Delta b)$ both equal to 2° . That’s fine for Δb , but you’d be sampling too closely in ℓ . That’s because the true (great-circle) angular distance between two points is $\Delta\ell \cdot \cos(b)$; there’s a foreshortening for $b \neq 0^\circ$ (e.g.: when you’re at the Earth’s North Pole, you can go completely around a small circle of constant b with just a few steps.). So you can save lots of observing time by observing at $\Delta\ell = \frac{2^\circ}{\cos(b)}$ instead of $\Delta\ell = 2^\circ$.

The true angular area is not the total extent in ℓ times the total extent in b because of the foreshortening discussed in the above paragraph. So the actual angular area of the Orion-Eridanus area isn’t the range in ℓ times the range in b , i.e. $60^\circ \times 60^\circ = 3600$ square degrees (which would require 900 pointings), but rather only about 2800 square degrees. At 2° sampling, that’s about 700 pointings.

These projects require what seems like a lot of pointings, but—hey!—the telescope is automated. With the magic of our computer-controlled telescope you can do these observations in just one or two days. But it will require planning and coordination with other groups—all of you are in the same boat! (Or should we say, “in the same dish”?) I suggest some trial observations to get started, both making sure you are observing correctly and also getting your datataking and reduction software in shape.

5. THE GALACTIC PLANE: THE GREAT CIRCLE AT LATITUDE ZERO ($b = 0^\circ$)

Point the telescope at a series of Galactic longitudes (ℓ) along the Galactic plane (Galactic latitude $b = 0^\circ$), over the full longitude range that is observable for our telescope. Getting nice results requires data over the largest section of the Galactic plane you can get, which is roughly (or somewhat less than) $\sim -10^\circ \lesssim \ell \lesssim 250^\circ$. This observation has two science goals:

1. *The Galactic rotation curve.* Determine the rotation curve of the Galaxy for the portion of the Galaxy inside the Sun and use the result to estimate the *gravitational* mass M_{grav} of the Galaxy that lies inside the Solar circle; for this, you just assume the usual $v^2 = \frac{GM_{grav}}{R}$. In fact, you can even get the radial dependence $M_{grav}(R)$! Also, use your observations to

estimate the *gaseous* mass $M_{gas}(R)$ of the Galaxy. What fraction of the total Galactic mass comes from interstellar gas? For this you'll need to know the radius of the Solar circle: it's 8.5 kpc. See *Commentary* below for a discussion.

2. *Spiral Structure of the Galaxy.* All your life, you've been told that we live in a spiral galaxy. Are they lying? In principle, to detect spiral structure you make a position-velocity plot along the Galactic plane [that's the image $T_A(\ell, V_{LSR})$ for $b = 0^\circ$]. Bright regions are regions of excess density—or regions in which lots of HI is packed into a narrow velocity range (see §14.1 below).

Do you see spirals in your position-velocity plot? How do you tell? There's only one easy way. Make a model of a spiral arm and see how it projects onto position-velocity space. To do this, you need to know the rotation curve. You've measured it for the inner Galaxy; for the outer Galaxy (outside the Solar circle) assume that the rotation curve is constant beyond the solar circle [$V(R)=220$ km/s for $R > 8.5$ kpc]. In polar coordinates, the equation for a spiral arm is

$$R_{arm} = R_0 e^{\kappa(\phi - \phi_0)} \tag{5}$$

where R_0 , κ , and ϕ_0 are free parameters; κ is the tangent of the *pitch angle*. You project this into position-velocity space using equation 12 and your knowledge of the Galactic geometry.

Comment: Finding spiral arms inside the Solar circle is very difficult, and has thwarted efforts by astronomers over the past several decades. Finding them in the outer Galaxy is not so bad.

6. IS THE GALACTIC PLANE FLAT?

External galaxies are often distorted (like the brim of a Fedora). These distortions are probably produced primarily by nearby neighbors. How about our own Galaxy? To determine this we want to look at the vertical structure of the Galactic plane and see where the peaks occur; a flat Galaxy would everywhere have the peaks at $b = 0^\circ$.

The most comprehensive way to observe and display these distortions is with a map that covers the biggest possible swath in Galactic longitude ℓ for the range of Galactic latitude (b) from -20° to $+20^\circ$. This is a huge area—almost 10^4 square degrees! At 2° sampling, you'd need ~ 2500 profiles. To make the project more manageable, you can increase the increment in ℓ , say to 4 degrees, while keeping the same, fully-sampled increment in b . Resolution in ℓ is less important because—like a fedora—the warp doesn't change rapidly with ℓ .

This project affords a great opportunity to make a spectacular color image in (ℓ, b) coordinates: brightness showing the amount of gas, color the Galactic rotation. Officially, the science goals here are to estimate the thicknesses of the Galactic disk and, also, to obtain an approximate

Fourier representation of the warp—which is not so straightforward given the absence of data in the “Southern” Galaxy.

7. THE GALACTIC POLES: GREAT CIRCLES AT CONSTANT LONGITUDE ℓ

How is the interstellar gas distributed within the Galaxy—for example, is it “disklike” or more spherically distributed? Are there any systematic motions other than just “Galactic rotation”? (You’d be surprised!). To answer these questions, determine the column density and mean velocity of interstellar gas as a function of Galactic latitude b .

To answer this completely you’d need to do a sky survey! That’s a big job. So let’s do something easier. We have one project (§5) that looks around a great circle, the Galactic equator (constant b). Other projects could check out one or more orthogonal great circles (at constant longitudes ℓ) going from one Galactic pole to the other and back again (or, if not the whole 360 degree circle, as much as possible). Particularly appropriate sets of constant-longitude great circles are the pairs $\ell = (220^\circ, 40^\circ)$ (partly because you can get the whole great circle) and $\ell = (130^\circ, 310^\circ)$ (because at positive latitudes there’s weak high-velocity gas).

The science goals here are to characterize the thickness of the gas layer and to determine the vertical kinematics of the gas, and if possible an estimate of the energy involved in the vertical motions.

8. MAPPING THE ORION-ERIDANUS SUPERBUBBLE

The “Orion-Eridanus Superbubble” was, and continues to be, produced by energetic stellar winds and supernovae that were located in immediate vicinity of the Orion nebula. For this, you’d map the region covering roughly ($\ell = 160^\circ \rightarrow 220^\circ, b = -70^\circ \rightarrow -10^\circ$). Interesting features should produce antenna temperatures ~ 20 K. This is almost 2800 square degrees and, with 2 degree spacing, requires almost 700 profiles. See §4!

The goal here is to map the HI in the 3-d space of (l, b, v) (that’s Galactic longitude, latitude, and velocity) and then—the challenging part—present the results in one or more color images to show the hollowed-out shell with the swept-up gas piled up at the edges.

9. MAPPING THE NORTH POLAR SPUR and its associated EXPANDING HI SHELL

This is a huge shell produced (and continuing) by several energetic stellar winds and supernovae that were located in the Sco/Oph association of stars. The shell contains not only HI but also

relativistic electrons, so the shell is visible in both the 21-cm line and synchrotron emission. We can't easily see the latter, but we can see the HI—and the pattern of its velocity follows that of an expanding shell.

For this, you'd map the region covering roughly ($\ell = 210^\circ \rightarrow 20^\circ, b = 0^\circ \rightarrow 90^\circ$). (The longitude range is $170^\circ: 210^\circ \rightarrow 360^\circ$ plus $0^\circ \rightarrow 20^\circ$). This is roughly 1/4 of the whole sky, so this is a huge area—almost 10000 square degrees! (This shell has an angular diameter of 120°). But (unfortunately) you can't measure the equivalent 2000 HI profiles because most of this area is too far south. At least, this makes the project do-able! In doing your observations, you need to cover as far south as you possibly can. See §4!

The goal here is to map the HI in the 3-d space of l, b, v (that's Galactic longitude, latitude, and velocity) and then—the challenging part—present the results in one or more color images to show the hollowed-out shell with the expanding swept-up gas piled up at the edges.

10. MAPPING A BIG HIGH-VELOCITY CLOUD

When we look up, away from the Galactic plane, we see infalling gas—some a very high velocities. It's called “High-Velocity Gas”. The line is weak (about 1 to 1.5 K), so you need high sensitivity—you need to use much longer integration times than for the above projects, at least a few minutes per point. This project needs to map the region bounded roughly by ($\ell = 60^\circ \rightarrow 180^\circ, b = 20^\circ \rightarrow 60^\circ$). This is about 3700 square degrees and needs about 900 profiles at 2° spacing. See §4!

The goal here is to map the HI in the 3-d space of l, b, v (that's Galactic longitude, latitude, and velocity) and then—the challenging part—present the results in one or more color images to show the hollowed-out shell with the expanding swept-up gas piled up at the edges.

11. MAPPING THE MAGELLANIC STREAM

The Magellanic Stream is an intergalactic tidal stream produced by gravitational interaction between the Magellanic Clouds and the Milky Way. The signal is weak (\sim a few tenths K), so you need to use much longer integration times than for the above projects—a minimum of 10 minutes per point. You need to map the pie-shaped area bounded roughly by ($\ell = 60^\circ \rightarrow 110^\circ, b = -90^\circ \rightarrow -30^\circ$). This is about 1250 square degrees and needs about 310 profiles at 2° spacing. See §4! Also, the LSR velocity of this gas ranges from roughly -400 to -100 km/s along its length, so you need frequency switch by a large enough interval!

The goal here is to map the HI in the 3-d space of l, b, v (that's Galactic longitude, latitude, and velocity) and then—the challenging part—present the results in one or more color images to show the hollowed-out shell with the expanding swept-up gas piled up at the edges.

12. DETECTING OH MASERS IN W49

The OH molecule is an important one for star formation and molecular clouds because it is one of the first to form when a diffuse atomic cloud starts to become denser; in other words, OH marks the transition between atomic and molecular gas. It persists at higher densities. The ground rotational/vibrational state exhibits “ Λ doubling”, which produces two spectral lines; each of these is hyperfine-split (just like the 21-cm line), and the net result is four lines centered at 1612.2310, 1665.4018, 1667.3590, 1720.5300 MHz. At sufficiently high densities, OH can be “pumped” by collisions, IR, or UV radiation to make these lines shine as masers.

The maser lines are strong, but our telescope is small, and the strongest source, W49, will produce < 1 K antenna temperature. We have interference, which makes the line difficult—maybe impossible—to detect. We’ve never detected it with our equipment.

The goal here is to detect the maser line in W49, which has $(ra, dec) = (19^h 10^m 17s, +09^\circ 06.0')$. This will require investigating the time/frequency properties of the interference using plots and images; excising the interference by hand; experimenting with algorithms to reduce or eliminate the interference (known as “signal processing”), presenting the results, and learning some of the basic maser theory.

We are currently working on this project to see if it is feasible. This project is more oriented towards computing and signal processing than “astronomy”, and as such would not appeal to everyone.

13. IF YOU HAVE TIME and ARE TOTALLY IN LOVE WITH RADIO ASTRONOMY (AND NOTHING OR NOBODY ELSE) THEN: MEASURE THE TELESCOPE BEAMWIDTH

Determine the beamwidth of the telescope by observing the Sun. Also answer the question: is the beam circularly symmetric? You’d think so, given that the telescope has a circular aperture. . .

Here you are using the Sun as a “point source” so that its antenna temperature T_A is proportional to the gain of the telescope as a function of angle. The resulting T_A is just the telescope’s diffraction pattern! The “half power beam width” (HPBW) is what radio astronomers generally use as the single-parameter specification of the angular size of the diffraction pattern.

Measure the HPBW and compare it with a theoretical estimate of the HPBW, which you can make from the usual diffraction argument.

14. BASICS OF THE 21-CM LINE.

14.1. Column Density and Mass

The 21-cm line, with frequency 1420.405751786 MHz, comes from atomic hydrogen (HI). In the terrestrial environment, H atoms quickly become H₂ molecules; in interstellar space, where densities are far lower than in the best vacuum systems on Earth and there are “lots” of UV photons that dissociate H₂, H remains atomic unless it resides inside dark clouds where it is shielded from starlight.

The intensity of the 21-cm line is directly proportional to the column density of H atoms as long as the opacity of the line is small; this is a reasonably good approximation (but not perfect, particularly in the Galactic plane where the line is strong). With this approximation,

$$N_{HI} = 1.8 \times 10^{18} \int T_B(v) dv \text{ cm}^{-2}. \quad (6)$$

Here N_{HI} is the column density of H atoms—the number of atoms in a 1 cm² column along the line of sight; $T_B(v)$ is the brightness temperature of the 21-cm line, which is a function of velocity v ; and v is the velocity in km s⁻¹. The velocity v is produced by the Doppler effect, so a frequency shift $\Delta\nu$ from line center corresponds to velocity $v = -c\frac{\Delta\nu}{\nu}$ or, for the 21-cm line, $v = -\frac{\Delta\nu}{4.73\text{kHz}}$. Note the minus sign! It means that positive velocities are receding—a very convenient convention for astronomy, because of the expansion of the Universe.

Note an important corollary to equation 6: If we consider a small velocity interval Δv , then the number of H atoms in the column in that velocity range is just

$$N_{HI}(v \rightarrow v + \Delta v) = 1.8 \times 10^{18} T_B(v) \Delta v \text{ cm}^{-2}. \quad (7)$$

This is very important, because it gives us the opportunity to make maps of the interstellar gas at different velocities and to determine how it moves.

To get the column density we need to get the brightness temperature T_B , which is not the same as our antenna temperature T_A . The relationship between these depends on the relative size of the source and the beam. This is equation (9) in the “Fount of All Knowledge!” handout, namely

$$T_A \approx T_B \frac{\Omega_s}{\Omega_s + \Omega_b}, \quad (8)$$

where Ω_s and Ω_b are the solid angles of the source and beam, respectively. From this, we see that if the source fills the beam ($\Omega_s \gtrsim \Omega_b$) then $T_A \approx \langle T_B \rangle$ (the brackets denote the average over solid angle); while if it is much smaller ($\Omega_s \ll \Omega_b$) then $T_A \approx \frac{\langle T_B \Omega_s \rangle}{\Omega_b}$, i.e. T_A is smaller by the ratio of the solid angles.

To get the mass of HI from the column density, you need to know the area of the region in *linear* measure, which is just Ωd^2 , where d is the distance. Each HI atom has mass m_H , so the total mass of HI in a region of size Ω is just

$$M_{HI} = m_H d^2 \langle N_{HI} \Omega \rangle . \quad (9a)$$

Here $\langle N_{HI} \Omega \rangle$ is shorthand for the general relation, which uses an integral:

$$\langle N_{HI} \Omega \rangle = \int_{\text{region}} N_{HI} d\Omega \quad (9b)$$

Substituting for N_{HI} from equation 7, we have

$$M_{HI}(v) = 1.8 \times 10^{18} \Delta v d^2 m_H \langle T_B \Omega \rangle \text{ gm} . \quad (10)$$

Units are cgs. Sometimes there is a well-defined source with boundaries; in this case, to get the mass of the whole source, we use the solid angle occupied by the source, Ω_s .

Sometimes (in fact, often) we want to know the mass seen *by the telescope* for some particular observation. For an extended region with $\Omega \gg \Omega_b$, $T_A = T_B$ and the telescope sees the solid angle Ω_b , so the product $\langle T_B \Omega \rangle = T_A \Omega_b$ because angular area seen by the telescope is limited by its beam size. In contrast, if the HI emission is limited to a small region—a small source with $\Omega_s \ll \Omega_b$ —then $T_A \approx \frac{\langle T_B \Omega_b \rangle}{\Omega_s}$ so the product $\langle T_B \Omega_s \rangle = T_A \Omega_b$; the telescope sees the whole source.

For both extremes, the product $T_B \Omega = T_A \Omega_b$. It's not only these two extremes, but for *all* sources the product $\langle T_B \Omega \rangle = T_A \Omega_b$. Thus, it is always true that the mass seen by the telescope is

$$M_{HI}(v) = 1.8 \times 10^{18} \Delta v d^2 m_H T_A \Omega_b \text{ gm} . \quad (11)$$

The distance d is usually a function of velocity v , especially in the Galactic plane.

$T_A(v)$ is directly measured and Ω_b is the telescope beam area, which is known, so it's easy to calculate $M_{HI}(v)$ —but only if you know the distance! Often you don't know the distance, but you might have a hunch for a reasonable value for the distance. Suppose this is 100 parsecs. In this case, people usually evaluate the mass for this distance and, when giving the mass, say “The mass is $xxx \times d_{100}$, where d_{100} is the distance in units of 100 pc”.

14.2. Converting Galactic Rotation to Doppler Velocity

Measurement of the Doppler shift caused by differential Galactic rotation for $0^\circ < \ell < 90^\circ$ allows a direct determination of the Galactic rotation curve inside the Solar circle, using the “tangent point” method. See Burton’s article “Structure of our Galaxy from Observations of HI” in *Galactic and Extragalactic Radio Astronomy, second edition* (editors: G.L. Verschuur and K.I. Kellermann). In essence, you use the following equation.

We won’t go through the derivation here; we cover it in class, and it is also done in Burton’s article, page 303-304. Let V_{Dopp} be the Doppler velocity (this is also the observed LSR velocity), $V(R)$ be the Galactic rotation velocity at Galactocentric distance R , and let the subscript \odot denote values at the Solar circle. Then we have

$$V_{Dopp} = \left[\frac{V(R)}{R} - \frac{V(R_\odot)}{R_\odot} \right] R_\odot \sin(\ell) \quad (12)$$

The values at the solar circle are $V(R_\odot) \approx 220$ km/s, $R_\odot \approx 8.5$ kpc.